CALCULUS
Linearity of the derivative, and derivatives of polynomials

NEW

0320-1. A car is traveling on a number line,
NEW
on which the unit of distance is a mile.
Its position at time t is $t^3 - 6t^2 + 9t - 3$,
with time measured in hours.

a. What is its velocity at time t, in miles/hr?

b. Graph its velocity, as a function of time.

c. When is its velocity equal to 0?

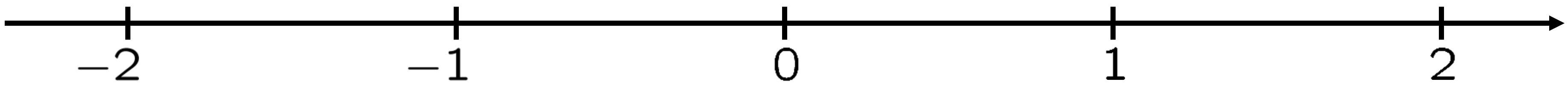
d. On what (maximal) intervals is the car moving in the positive direction?

e. On what (maximal) intervals is the car moving in the negative direction?

f. On what (maximal) intervals is the car's acceleration positive?

g. On what (maximal) intervals is the car's acceleration negative?

0320-2. A particle is traveling on a number line
NEW
The positive direction is to the right, *viz.*:



The position of the particle, at time t,
is $-4t^2 + 8t + 3$.

a. What is its velocity at time t?

b. When is its velocity equal to 0?

c. On what (maximal) intervals is the particle moving to the left?

d. On what (maximal) intervals is the particle moving to the right?

e. At what time is the particle farthest right?

f. What is its maximial (*i.e.*, rightmost) position?

0320-3. A rock is thrown on the moon.
NEW
Its initial velocity (straight upward) is
18 meters/second.
Its height above the lunar surface, t seconds after release, is

$$h(t) \doteq -(0.82)t^2 + 18t + 1.1,$$

in meters.

a. What is its velocity at time t, in meters per second?

b. When is its velocity equal to 0?

c. For how long a time (in seconds), after release, is the rock moving upward?

d. What is the maximal height above the lunar surface reached by the rock, in meters?

0320-4. A square is growing
NEW
in such a way that its side length at time t is equal to $2t + 4$.

Its enclosed area is (side length)^2, and its circumference is 4(side length).

a. Find a formula for its enclosed area at time t.

b. Find a formula for the rate of change in its enclosed area at time t.

c. Find a formula for its circumference at time t.

0320-5. A circle is growing
NEW
in such a way that its diameter at time t is equal to $2t+4$.

Its enclosed area is $\pi(\text{radius})^2$, and its circumference is $2\pi(\text{radius})$.

a. Find a formula for its enclosed area at time t.

b. Find a formula for the rate of change in its enclosed area at time t.

c. Find a formula for its circumference at time t.

0320-6. NEW The gravitational force (in newtons) exerted by the earth on the moon is given by the formula $F \doteq (2.93 \times 10^{37})/r^2$, where r is their distance apart in km.

a. If the distance increases from 382,000 km to 384,000 km then what is the corresponding change in force (in newtons)? That is, compute $[F]_{r:\to 382000}^{r:\to 384000}$.

b. Compute the difference quotient $\left([F]_{r:\to 382000}^{r:\to 384000}\right)/2000$.

c. Compute $[dF/dr]_{r:\to 383000}$.

0320-7. The speed of sound (in meters/sec) is
NEW
$c \doteq 20\sqrt{\theta + 273}$, where θ is the air temperature (in Celcius).

a. If the air temperature increases from 8° Celcius to 10° Celcius, then what is the corresponding change in the speed of sound (in meters/second)?

That is, compute $[c]_{\theta:\to 8}^{\theta:\to 10}$.

b. Compute the difference quotient

$$\left([c]_{\theta:\to 8}^{\theta:\to 10}\right)/2.$$

c. Compute $[dc/d\theta]_{\theta:\to 9}$.

0320-8. We study the populations of two species, wolves and sheep, on a certain plot of land.

NEW

Let S be the number of sheep at time t and let W be the number of wolves at time t.

We model the population counts as follows:

$$dW/dt = 2S + 5W - 116$$
$$dS/dt = \quad S + 3W - 67$$

At what counts, W and S, will the population be stable?

(Stability means: $dW/dt = 0 = dS/dt$.)

0320-9. The position of a particle along a number line is given by

NEW

$$p(t) = (0.04)t^7 - (0.002)t^6 + (0.9)t^5 - 8t^4 - 5t^3 + t^2 + t + 1.$$

Compute its velocity, acceleration, jerk, snap, crackle and pop at time t.