CALCULUS
The Mean Value Theorem
NEW

0460-1. Let $f(x) = x^2 - 2x - 3$.
NEW

a. Check that f satisfies the conditions of Rolle's Theorem on the interval $[-1, 3]$.
That is, check

(i) that f is continuous on $[-1, 3]$,
(ii) that f is differentiable on $(-1, 3)$
and (iii) that $f(-1) = f(3)$.

b. Find all solutions to the equation in the conclusion of Rolle's Th'm for f on $[-1, 3]$.
That is, find all $c \in (-1, 3)$ s.t. $f'(c) = 0$.

0460-2. Let $f(x) = x^2 + x - 3$.
NEW

a. Check that f satisfies the conditions of the MVT on the interval $[-1, 3]$.
That is, check

(i) that f is continuous on $[-1, 3]$
and (ii) that f is differentiable on $(-1, 3)$.

b. Find all solutions to the equation in the conclusion of the MVT for f on $[-1, 3]$.
That is, find all $c \in (-1, 3)$ s.t.

$$f'(c) = \frac{[f(3)] - [f(-1)]}{3 - (-1)}.$$

0460-3. Let $f(x) = 7 + |x - 5|$.
NEW

a. Show that f is continuous on $[2, 8]$.

b. Show that $f(2) = f(8)$.

c. Show that the conclusion of Rolle's Th'm, for f on $[2, 8]$, fails. That is, show that there is no $c \in (2, 8)$ s.t. $f'(c) = 0$.

d. Explain why this does not contradict Rolle's Theorem.

0460-4. Let $f(x) = x + |x - 5|$.
NEW

a. Show that f is continuous on $[2, 8]$.

b. Show that the conclusion of the MVT, for f on $[2, 8]$, fails. That is, show that there is no $c \in (2, 8)$ s.t.

$$f'(c) = \frac{[f(8)] - [f(2)]}{8 - 2}.$$

c. Explain why this does not contradict the MVT.

0460-5. NEW Let $f(x) = \begin{cases} 100, & \text{if } x = 2 \\ 3x - 5, & \text{if } 2 < x < 8 \\ 40, & \text{if } x = 8. \end{cases}$

a. Show that f is differentiable on $(2, 8)$.

b. Show that the conclusion of the MVT, for f on $[2, 8]$, fails. That is, show that there is no $c \in (2, 8)$ s.t.

$$f'(c) = \frac{[f(8)] - [f(2)]}{8 - 2}.$$

c. Explain why this does not contradict the MVT.

0460-6. NEW Show that $3x + \cos(2x) = 100$ has exactly one real solution.

0460-7. NEW Let c be any constant.

Show that $x^3 + x + c = 0$ has at most one real solution on $\mathbb{R}$.

0460-8. NEW At noon on some day, a certain car is at the 200 mile marker on some road. The speed limit on the road is 55 mph. A driver drives the car for seven hours, obeying the speed limit.

Let $f(t)$ denote the position of the car t hours after noon; then

$$f(0) = 200$$

and

$$\forall t \in [0,7], \quad f'(t) \leq 55.$$

With these constraints, what is the largest possible value for $f(7)$?