

CALCULUS
Volume by slices and
the disk and washer methods:
Problems
NEW

0720-1. NEW Let R be the region bounded by
 $y = x + 2$ and $x = 3$ in $2 \leq y \leq 3$.

- Sketch R .
- Find the volume of the solid obtained by rotating R about the x -axis.
- Find the volume of the solid obtained by rotating R about the y -axis.

0720-2. NEW Let R be the region bounded by
 $y = x^2$ and $y = x + 6$.

- Sketch R .
- Find the volume of the solid obtained by rotating R about the x -axis.
- Find the volume of the solid obtained by rotating R about the line $x = -3$.

0720-3. Let R be the region bounded by
 $y = \ln x$, $x = 7$ and $y = 1$.

- Sketch R .
- Find the volume of the solid obtained by rotating R about the y -axis.

0720-4. Let R be the region bounded by
 $y = \sin x$ and $y = 0$ in $0 \leq x \leq \frac{\pi}{4}$.

- Sketch R .
- Find the volume of the solid obtained by rotating R about the x -axis.

Hint: $\sin^2 x = \frac{1 - [\cos(2x)]}{2}$

0720-5. NEW Let R be the region bounded by

$$(x + 1)^2 + (y - 4)^2 = 9.$$

- a. Sketch R .
- b. Find the volume of the solid obtained by rotating R about the x -axis.

Note: This solid is called a torus. It is in the shape of a doughnut.

Hint: Remember that $2 \int_{-3}^3 \sqrt{9 - u^2} dx$ is known; it is the area enclosed in a circle of radius 3.

NEW 0720-6. Let R be the region bounded by

$$y = x^2 \text{ and } x = y^3.$$

- Sketch R .
- Find the volume of the solid obtained by rotating R about the line $y = -1/3$.
- Find the volume of the solid obtained by rotating R about the line $x = -1/2$.

NEW 0720-7. Let R be the region bounded by

$$y = x^2 \text{ and } x = y^2.$$

- Sketch R .
- Find the volume of the solid obtained by rotating R about the line $y = -1/3$.
- Find the volume of the solid obtained by rotating R about the line $x = -1/2$.

0720-8. Let R be the region bounded by
 $x = \sin y$, $x = \cos y$ in $0 \leq y \leq \pi/5$.

Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating R about the line $x = -3$.

0720-9. Describe the solid of revolution whose volume is given by

$$\pi \int_3^5 (4e^{8y} - 9 \cos^2 y) dy.$$

Do not evaluate this integral.

0720-10. Describe the solid of revolution whose volume is given by

$$\pi \int_0^\pi (2 + \sin y)^2 - 4 dy.$$

Do not evaluate this integral.

0720-11. A solid S sits above a horizontal plane P . $\forall x \geq 0$, let P_x be the horizontal plane that is x units above P . Suppose that S lies between P_1 and P_2 . Suppose, also, that $\forall x \in [1, 2]$, the intersection of S and P_x is the region inside a trapezoid

whose base has length $5x$

whose top has length $7x$

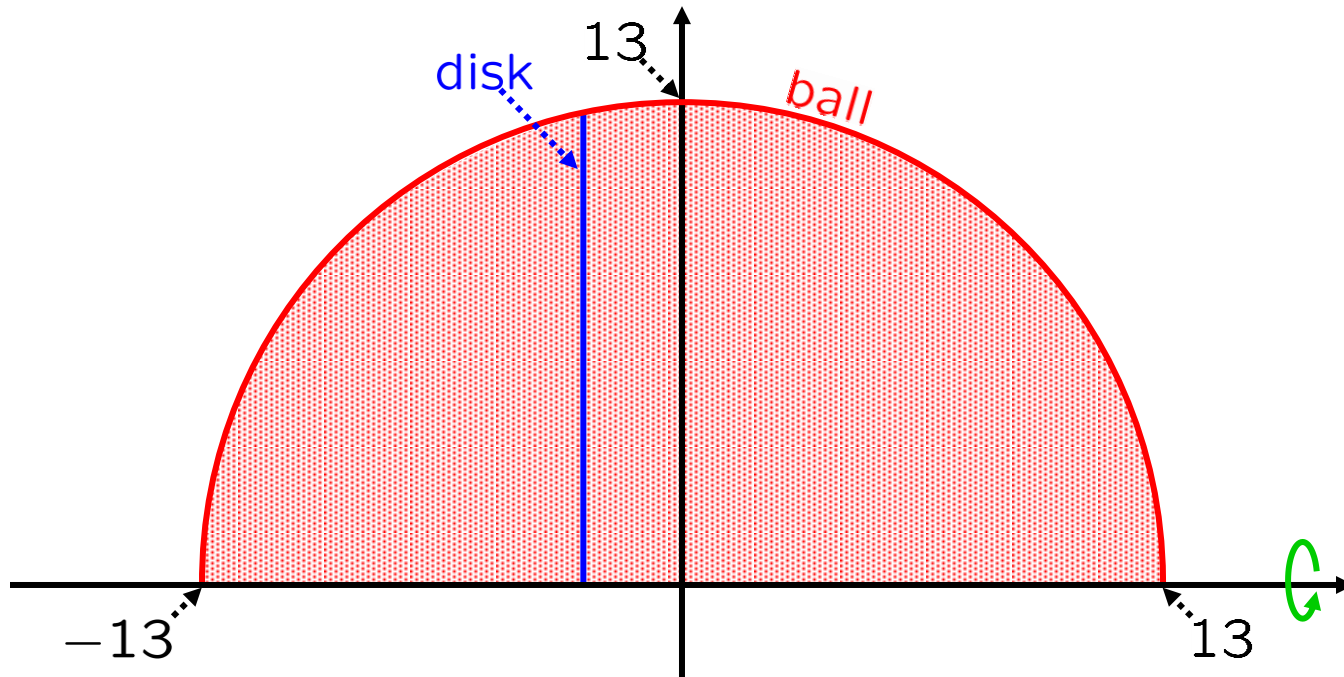
and whose altitude has length e^{3x^2} .

Compute the volume of S .

Hint: Remember that if b , t and a are the lengths of the base, top and altitude of a trapezoid,

then the area inside it is $a(b + t)/2$.

0720-12. Using the disk method, find the volume in a ball of radius 13, following the diagram shown below.



0720-13. We create a napkin holder by drilling a cylindrical hole of radius 5 through the middle of a ball of radius 13, as shown below. Using the washer method, find its volume.

