CALCULUS
The derivative of a function is a function
NEW
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

Choose red, green or purple.
The graph of \( f \) is shown above. Which of the following is the graph of \( f' \)?

**ANSWER:**

Choose red, **green** or purple.
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

Choose red, green or purple.
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

**Answer:** Choose red, green or purple.
The graph of $f$ is shown above. Freehand a sketch of the graph of $f'$. On your graph, indicate 1 and $-1$ on the horizontal axis.
ANSWER:
The graph of $f$ is shown above.

a. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not defined?

b. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not continuous?

c. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not differentiable?
The graph of \( f \) is shown above.

a. At which of the numbers \(-3, -2, -1, 0, 1, 2, 3\) is \( f \) not defined?  \textbf{ANS:} \(-3, -2\)

b. At which of the numbers \(-3, -2, -1, 0, 1, 2, 3\) is \( f \) not continuous?  \textbf{ANS:} \(-3, -2, 1, 2\)

c. At which of the numbers \(-3, -2, -1, 0, 1, 2, 3\) is \( f \) not differentiable?  \textbf{ANS:} \(-3, -2, 1, 2, 3\)
The graphs of $f$, $f'$ and $f''$ are shown above. Which is which?

State the color of $f$, the color of $f'$ and the color of $f''$. 
The graphs of $f$, $f'$ and $f''$ are shown above. Which is which?

State the color of $f$, the color of $f'$ and the color of $f''$. 
0280-6. Let \( f(t) = 2t^3 + 4t \).

a. What is the domain of \( f \)?

b. Using the definition of the derivative, and using the cubic binomial formula
\[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3,\]
compute \( f'(t) \).

c. What is the domain of the derivative \( f' \)?
0280-6. Let \( f(t) = 2t^3 + 4t \).

a. **What** is the domain of \( f \)?

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**ANSWER:**

a. \( \mathbb{R} = (-\infty, \infty) \)
0280-6. Let \( f(t) = 2t^3 + 4t \).

b. Using the definition of the derivative, and using the cubic binomial formula
\[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3,\]
compute \( f'(t) \).

**ANSWER:**

\[ f'(t) = \lim_{h \to 0} \frac{[2(t + h)^3 + 4(t + h)] - [2t^3 + 4t]}{h} \]

\[ = \lim_{h \to 0} \frac{2(t^3 + 3t^2h + 3th^2 + h^3) + 4(t + h)] - [2t^3 + 4t]}{h} \]

\[ = \lim_{h \to 0} \frac{2(3t^2h + 3th^2 + h^3) + 4h}{h} \]

\[ = \lim_{h \to 0} 2(3t^2 + 3th + h^2) + 4 \]

\[ = 2(3t^2) + 4 = 6t^2 + 4 \]
0280-6. Let \( f(t) = 2t^3 + 4t \).

c. What is the domain of the derivative \( f' \)?

**ANSWER:**

\[
f'(s) = 6t^2 + 4
\]

c. \( \mathbb{R} = (-\infty, \infty) \)
0280-7. Let \( f(z) = \frac{1 - 3z}{2 + 5z} \).

a. What is the domain of \( f \)?

b. Using the definition of the derivative, compute \( f'(z) \).

c. What is the domain of the derivative \( f' \)?
0280-7. Let \( f(z) = \frac{1 - 3z}{2 + 5z} \).

a. What is the domain of \( f \)?

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**ANSWER:**

a. \( \mathbb{R} \setminus \{-2/5\} \)
Let \( f(z) = \frac{1 - 3z}{2 + 5z} \).

b. Using the definition of the derivative, compute \( f'(z) \).

\[
\text{ANS: } \frac{(f(z + h)) - (f(z))}{h} = \frac{1}{h} \left[ \frac{1 - 3z - 3h}{2 + 5z + 5h} - \frac{1 - 3z}{2 + 5z} \right]
\]

\[
= \frac{1}{h} \left[ \frac{(1 - 3z - 3h)(2 + 5z) - (2 + 5z + 5h)(1 - 3z)}{(2 + 5z + 5h)(2 + 5z)} \right]
\]

\[
= \frac{-3h(2 + 5z) - 5h(1 - 3z)}{h(2 + 5z + 5h)(2 + 5z)}
\]

\[
= \frac{-11h}{h(2 + 5z + 5h)(2 + 5z)} \quad h \to 0 \quad \Rightarrow \quad \frac{-11}{(2 + 5z)^2}
\]
Let \( f(z) = \frac{1 - 3z}{2 + 5z} \).

c. What is the domain of the derivative \( f' \)?

**ANSWER:** \[ f'(x) = \frac{-11}{(2 + 5z)^2} \]

\[ \mathbb{R} \setminus \{-2/5\} \]
0280-8. Let $g(x) = |x^2 - 2x + 1|$.

At which numbers is $g$ not differentiable?

**Hint:** Determine the (maximal) intervals where $x^2 - 2x + 1$ is positive and negative.

Sketch the graph of $y = x^2 - 2x + 1$.

Sketch the graph of $y = g(x)$.

**GENERAL RULE:**
At numbers $x$ where $x^2 - 2x + 1$ has a root of multiplicity one, $g$ is not differentiable. Everywhere else, $g$ is differentiable.
Let \( g(x) = |x^2 - 2x + 1| \).

At which numbers is \( g \) not differentiable?

**ANSWER:** \( x^2 - 2x + 1 = (x - 1)^2 \)

positive on \( x \neq 1 \)
Let \( g(x) = |x^2 - 2x + 1| \).

At which numbers is \( g \) not differentiable?

**ANSWER:** \( x^2 - 2x + 1 = (x - 1)^2 \) is positive on \( x \neq 1 \).

\[ y = |x^2 - 2x + 1| \]

\( g \) is differentiable at every real number.

**GENERAL RULE:**
At numbers \( x \) where \( x^2 - 2x + 1 \) has a root of multiplicity one, \( g \) is not differentiable. Everywhere else, \( g \) is differentiable.
0280-9. Let \( f(x) = |x^5 - 2x^4 + x^3| \).

At which numbers is \( f \) not differentiable?

**Hint:**
\[
y = x^5 - 2x^4 + x^3 \text{ is hard to graph, but you don’t have to; just use the...}
\]

**GENERAL RULE:**
At numbers \( x \) where \( x^5 - 2x^4 + x^3 \) has a root of multiplicity one, \( f \) is not differentiable. Everywhere else, \( f \) is differentiable.
Let \( f(x) = |x^5 - 2x^4 + x^3| \).

At which numbers is \( f \) not differentiable?

**ANSWER:** \( x^5 - 2x^4 + x^3 = x^3(x - 1)^2 \)

\( x^5 - 2x^4 + x^3 \) has no roots of multiplicity one, so \( f \) is differentiable at every real number.

**GENERAL RULE:**
At numbers \( x \) where \( x^5 - 2x^4 + x^3 \) has a root of multiplicity one, \( f \) is not differentiable. Everywhere else, \( f \) is differentiable.