

# CALCULUS

## Derivatives and rates of change

### NEW

**WARNING:** In this homework, derivatives must be computed from the definition, *i.e.*, as the limit of the difference quotient. Do NOT use product, quotient or chain rules, or any other technique coming from a later topic.

0270-1. NEW Let  $C$  be the curve  $y = -x^2 + 2x + 3$ .

Let  $L$  be the tangent line to  $C$  at the point  $(1, 4)$ .

- Find the slope of  $L$ , by computing a limit of slopes of secant lines.
- Find an equation of  $L$ .
- Graph  $C$  and  $L$  in the rectangle  $-2 \leq x \leq 4$ ,  $-2 \leq y \leq 5$ .
- Graph  $C$  and  $L$  in the rectangle  $0 \leq x \leq 2$ ,  $1 \leq y \leq 5$ .
- Graph  $C$  and  $L$  in the rectangle  $0.9 \leq x \leq 1.1$ ,  $3.8 \leq y \leq 4.2$ .

In **c**, **d** and **e**, note that, as you “zoom in”, the tangent line looks more and more like the curve.

0270-2.  
NEW

a. Compute  $\lim_{h \rightarrow 0} \frac{\sqrt{9 + 2h} - 3}{h}$ .

- b. Find the slope of the secant line to  $y = \sqrt{2x + 1}$  through the points  $(4, 3)$  and  $(4 + h, \sqrt{9 + 2h})$ .
- c. Find an equation of the tangent line to  $y = \sqrt{2x + 1}$  at the point  $(4, 3)$ .

0270-3.  
NEW

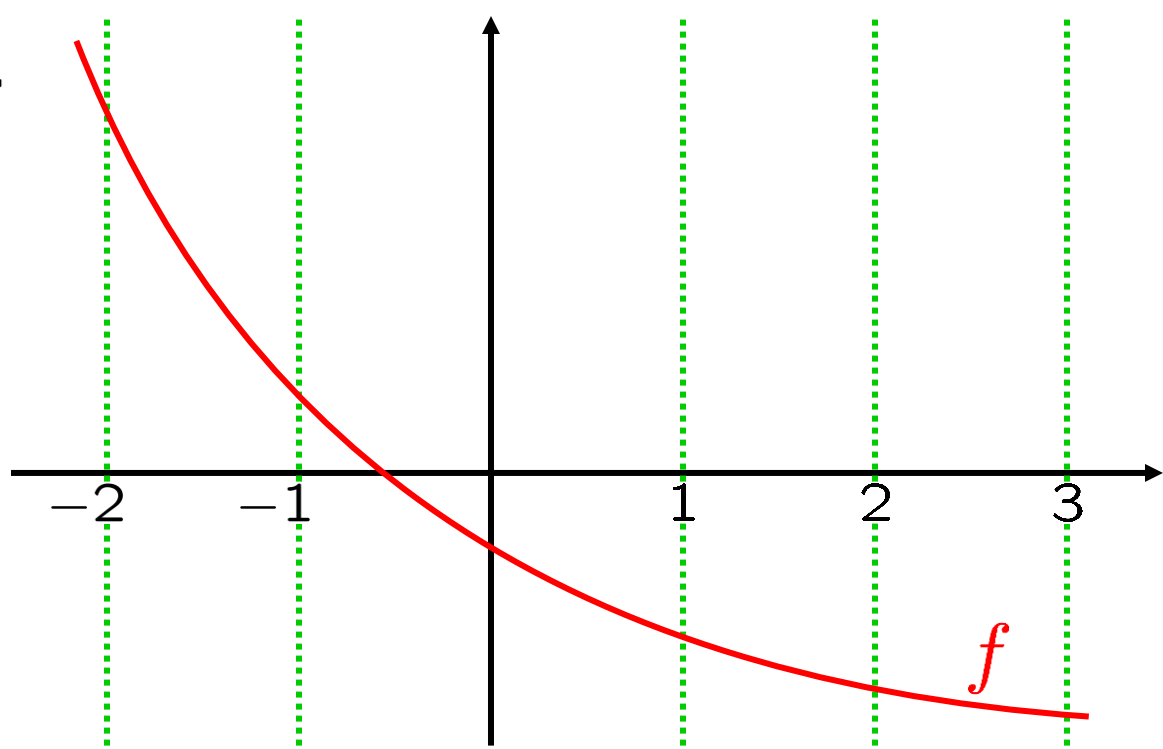
A particle moves on a number line. Its position at any time  $t$  is  $\sqrt{2t + 1}$ .

- a. Find the average velocity between time  $t = 4$  and time  $t = 4 + h$ .
- b. Find the instantaneous velocity at time  $t = 4$ .

0270-4. A heavy object is taken to the top of a building 250 feet high. At time  $t = 0$ , it is thrown upward at 40 feet/second. We engage the services of two Nobel prize-winning physicists who confer (*i.e.*, yell and scream at one another). After several hours of scholarly study, followed by minor medical treatment for blunt trauma, lacerations and contusions, they hold a joint press conference, and inform their public that,  $t$  seconds after release, the object will be located

$$250 + 40t - 16t^2 \quad \text{feet}$$

above the ground. Based on this, **find** the the velocity of the object 0.1 seconds after release. **Give** your answer in feet per second.



Order these numbers, from smallest to largest:

$$f'(-2), f'(-1), f'(0), f'(1), f'(2), f'(3)$$

Note that we are asking about  $f'$ , **not**  $f$ .



0270-6. NEW Let  $f(x) = \frac{9x - 2}{5x - 1}$ .

Do NOT use the quotient rule.  
Use only the definition of the  
derivative as the limit of the  
difference quotient.

a. Compute  $f'(2)$ .

b. Compute  $f'(3)$ .

c. Compute  $f'(4)$ .

d. Compute  $f'(a)$ , for an arbitrary number  $a$ .

0270-7. Find a function  $f$  and a number  $a$  s.t.

$$f'(a) = \lim_{h \rightarrow 0} \frac{[\arctan(-3 + h)] - [\arctan(-3)]}{h}.$$