

CALCULUS
Implicit differentiation
NEW

0430-1. ^{NEW} Let an expression y of x be given, implicitly, by the formula $x^2y + \pi x - \sqrt{2}y = 3$.

- a. Find dy/dx by implicit differentiation.
- b. Solve for y as an explicit expression of x .
- c. Differentiate your answer to Part b, writing dy/dx as an explicit expression of x .
- d. Substitute your answer for Part b into every y appearing in your answer to Part a, writing dy/dx as an explicit expression of x .
- e. Verify that your answers to Part c and Part d are the same.

0430-2. **NEW** Let an expression y of x be given, implicitly, by the formula $x^4 + y^3 = 1$.

- a. **Find** dy/dx by implicit differentiation.
- b. **Solve** for y as an explicit expression of x .
- c. **Differentiate** your answer to Part b, writing dy/dx as an explicit expression of x .
- d. **Substitute** your answer for Part b into **every** y appearing in your answer to Part a, writing dy/dx as an explicit expression of x .
- e. **Verify** that your answers to Part c and Part d are the same.

NEW 0430-3. Let an expression y of x be given, implicitly, by the formula

$$ye^x - \sqrt{2}e^{2x} + e^y \csc x = 2.$$

Find dy/dx by implicit differentiation.

NEW 0430-4. Let an expression y of x be given, implicitly, by the formula

$$\cos^2 y = -x + \sqrt[3]{7}y.$$

Find dy/dx by implicit differentiation.

0430-5. **NEW** Let an expression y of x be given, implicitly, by the formula

$$x^4 + y^3 = 15.$$

Find an equation of the tangent line to the graph of this equation at the point $(-2, -1)$.

0430-6. **NEW** Let an expression y of x be given, implicitly, by the formula

$$3y^2 = x^6 - \sqrt[3]{16x}.$$

Find an equation of the tangent line to the graph of this equation at the point $(\sqrt[3]{4}, 2)$.

0430-7. NEW Let an expression y of x be given, implicitly, by the formula

$$-x^4 + \sqrt[7]{2}y^3 + e^2y = 2.$$

Find d^2y/dx^2 by implicit differentiation.

0430-8. NEW Let an expression y of x be given, implicitly, by the formula

$$-x^7 + 4\pi y^3 = 8 + xy.$$

Find d^2y/dx^2 by implicit differentiation.

0430-9. For every $a \in \mathbb{R}$, for every $b > 0$,
NEW let G_a be graph of $15x^3 - 8y^2 = ax^3y^2$ and
let H_b be graph of $x^5 + y^4 = b$.

a. Let p be the point $(1, 1)$, which lies
both on G_7 and on H_2 .

Show that the tangent lines to G_7 and H_2
at p are perpendicular.

b. Let a and b be any two real numbers,
with $b > 0$.

Let q be any point which lies
both on G_a and on H_b .

Show that the tangent lines to G_a and H_b
at q are perpendicular.

Challenge problem (not assigned):

N/A

For every $a, b \in \mathbb{R}$,

let G_a be graph of $e^{-x} - e^{-y} = a$ and

let H_b be graph of $e^x + e^y = 2b$.

a. Let p be the point $(1, 1)$, which lies both on G_0 and on H_e .

Show that the tangent lines to G_0 and H_e at p are perpendicular.

b. Let a and b be any two real numbers.

Let q be any point which lies both on G_a and on H_b .

Show that the tangent lines to G_a and H_b at q are perpendicular.