CALCULUS
The Mean Value Theorem
NEW

0460-1. Let $f(x) = x^2 - 2x + 3$.
NEW

a. Check that f satisfies the conditions of Rolle's Theorem on the interval $[0, 2]$. That is, check

(i) that f is continuous on $[0, 2]$,

(ii) that f is differentiable on $(0, 2)$

and (iii) that $f(0) = f(2)$.

b. Find all solutions to the equation in the conclusion of Rolle's Th'm for f on $[0, 2]$. That is, find all $c \in (0, 2)$ s.t. $f'(c) = 0$.

0460-2. Let $f(x) = x^2 + x + 3$.
NEW

a. Check that f satisfies the conditions of the MVT on the interval $[0, 2]$.
That is, check

(i) that f is continuous on $[0, 2]$

and (ii) that f is differentiable on $(0, 2)$.

b. Find all solutions to the equation in the conclusion of the MVT for f on $[0, 2]$.
That is, find all $c \in (0, 2)$ s.t.

$$f'(c) = \frac{[f(2)] - [f(0)]}{2 - 0}.$$

NEW 0460-3. Let $f(x) = 3 + 2|x - 5|$.

a. Show that f is continuous on $[1, 9]$.

b. Show that $f(1) = f(9)$.

c. Show that the conclusion of Rolle's Th'm, for f on $[1, 9]$, fails. That is, show that there is no $c \in (1, 9)$ s.t. $f'(c) = 0$.

d. Explain why this does not contradict Rolle's Theorem.

0460-4. Let $f(x) = -x + 2|x - 5|$.
NEW

a. Show that f is continuous on $[1, 9]$.

b. Show that the conclusion of the MVT, for f on $[1, 9]$, fails. That is, show that there is no $c \in (1, 9)$ s.t.

$$f'(c) = \frac{[f(9)] - [f(1)]}{9 - 1}.$$

c. Explain why this does not contradict the MVT.

0460-5. NEW Let $f(x) = \begin{cases} 398, & \text{if } x = 1 \\ 3x - 5, & \text{if } 1 < x \leq 9 \end{cases}$

a. Show that f is differentiable on $(1, 9)$.

b. Show that the conclusion of the MVT, for f on $[1, 9]$, fails. That is, show that there is no $c \in (1, 9)$ s.t.

$$f'(c) = \frac{[f(9)] - [f(1)]}{9 - 1}.$$

c. Explain why this does not contradict the MVT.

0460-6. Show that $xe^{x^2} = -5$ has exactly one real solution.
NEW

0460-7. Let c be any constant.
NEW

Show that $e^{x^3} + x + c = 0$ has at most one real solution on $\mathbb{R}$.

0460-8. NEW At noon on some day, a certain car is at the 250 mile marker on some road. The speed limit on the road is 50 mph. A driver drives the car for nine hours, obeying the speed limit.

Let $f(t)$ denote the position of the car t hours after noon; then

$$f(0) = 250$$

and

$$\forall t \in [0, 9], \quad f'(t) \leq 50.$$

With these constraints, what is the largest possible value for $f(9)$?