CALCULUS

Definite integration and Riemann sum problems

NEW

0590-1. NEW Let $f(x) = 3 + x^2$.

a. Compute $L_3S_{-1}^1 f$.
Sketch f over $[-1, 1]$ and add,
into your sketch, the three
rectangles represented by $L_3S_{-1}^1 f$.

b. Compute $M_3S_{-1}^1 f$.
Sketch f over $[-1, 1]$ and add,
into your sketch, the three
rectangles represented by $M_3S_{-1}^1 f$.

c. Compute $R_3S_{-1}^1 f$.
Sketch f over $[-1, 1]$ and add,
into your sketch, the three
rectangles represented by $R_3S_{-1}^1 f$.

NEW 0590-2. Let $f(x) = e^x$.

a. Compute $L_4 S_0^8 f$ to three decimal places.

b. Compute $M_4 S_0^8 f$ to three decimal places.

c. Compute $R_4 S_0^8 f$ to three decimal places.

NEW 0590-3. Let $f(x) = \sin^2 x$.

a. Compute $L_3 S_0^\pi f$ to three decimal places.

b. Compute $M_3 S_0^\pi f$ to three decimal places.

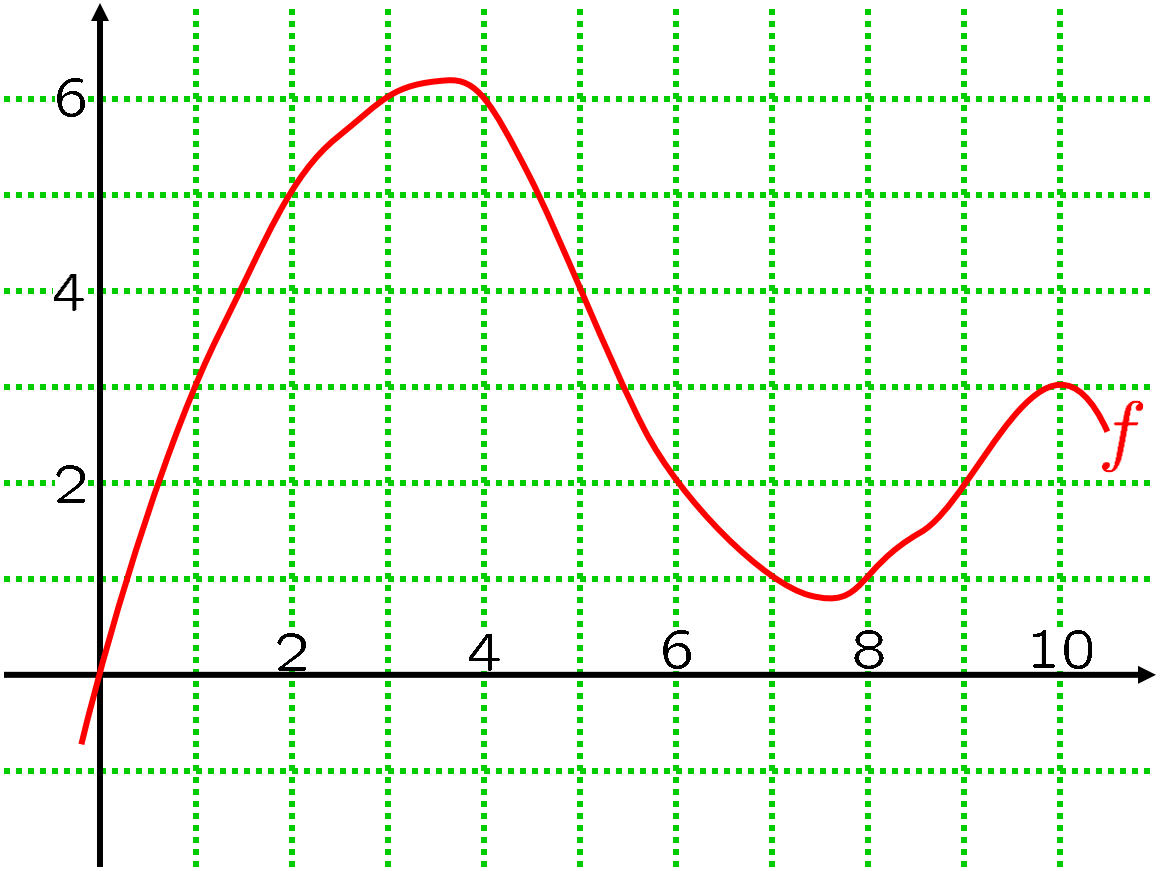
c. Compute $R_3 S_0^\pi f$ to three decimal places.

NEW 0590-4. A car's acceleration is positive from time 0 to time 30 seconds, and its velocity at various times is given in the table below.

time (secs)	0	5	10	15	20	25	30
velocity (ft/sec)	0	50	75	80	81	82	83

Find upper and lower estimates for the distance traveled by the car over these 30 seconds.

0590-5. NEW The gph of a function f appears below.



Estimate $\int_0^{10} f(x)\,dx$ by computing

(a) $L_5 S_0^{10} f$, (b) $M_5 S_0^{10} f$ and (c) $R_5 S_0^{10} f$.

0590-6. NEW Express the area under $y = e^{-x^2/3}$ from $x = -2$ to $x = 2$ as a limit of midpoint Riemann sums. (Don't evaluate the limit.)

0590-7. NEW Express the area under $y = \sqrt{x^3 + x + 9}$ from $x = 1$ to $x = 4$ as a limit of left endpoint Riemann sums. (Don't evaluate the limit.)

0590-8. NEW Express the area under $y = \cos\left(x^3 - x\right)$ from $x = -3$ to $x = 6$ as a limit of right endpt Riemann sums. (Don't evaluate the limit.)

NEW 0590-9. Express $\displaystyle\int_{-2}^{3} \frac{e^{-x^2}}{\sqrt{\pi}}\,dx$ as a limit of midpoint Riemann sums. (Don't evaluate the limit.)

NEW 0590-10. Let $f(x) = x^3 + x$.

a. Write $R_n S_0^2 f$ as a rational expression in n (*i.e.*, as one polynomial in n divided by another).

b. Compute $\lim_{n\to\infty} R_n S_0^2 f$.

0590-11. NEW The limit

$$\lim_{n\to\infty}\left[\frac{7}{n}\sum_{j=0}^{n-1}\left(\sin^2\left(4+j(7/n)\right)\right)\right]$$

represents the area under $y=f(x)$ from $x=a$ to $x=b$, for some choice of $f(x)$, a and b.

a. Find $f(x)$, a and b.

b. Express the limit as a definite integral.

0590-12. NEW The limit

$$\lim_{n\to\infty}\left[\frac{6}{n}\sum_{j=1}^{n}\left(\sin\left(\frac{1}{2+j(6/n)}\right)\right)\right]$$

represents the area under $y = f(x)$ from $x = a$ to $x = b$, for some choice of $f(x)$, a and b.

a. Find $f(x)$, a and b.

b. Express the limit as a definite integral.

0590-13. NEW Let $f(x) = 4 - \sqrt{9 - x^2}$.

a. Sketch the graph of $y = f(x)$.

b. Compute $\int_{-3}^{3} f(x)\,dx$, by interpreting this integral as an area.