

CALCULUS

Derivatives and rates of change

OLD

WARNING: In this homework, derivatives must be computed from the definition, *i.e.*, as the limit of the difference quotient. Do NOT use product, quotient or chain rules, or any other technique coming from a later topic.

0270-1. Let C be the curve $y = x^2 - 3x + 5$.

OLD

Let L be the tangent line to C at the point $(2, 3)$.

- Find the slope of L , by computing a limit of slopes of secant lines.
- Find an equation of L .
- Graph C and L in the rectangle
 $-1 \leq x \leq 4, \quad -1 \leq y \leq 6$.
- Graph C and L in the rectangle
 $1 \leq x \leq 3, \quad 2 \leq y \leq 6$.
- Graph C and L in the rectangle
 $1.9 \leq x \leq 2.1, \quad 2.8 \leq y \leq 3.2$.

In c, d and e, note that, as you “zoom in”, the tangent line looks more and more like the curve.

0270-2.
OLD

a. Compute $\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$.

b. Find the slope of the secant line to $y = \sqrt{x+1}$ through the points $(3, 2)$ and $(3+h, \sqrt{4+h})$.

c. Find an equation of the tangent line to $y = \sqrt{x+1}$ at the point $(3, 2)$.

0270-3.
OLD

A particle moves on a number line. Its position at any time t is $\sqrt{t+1}$.

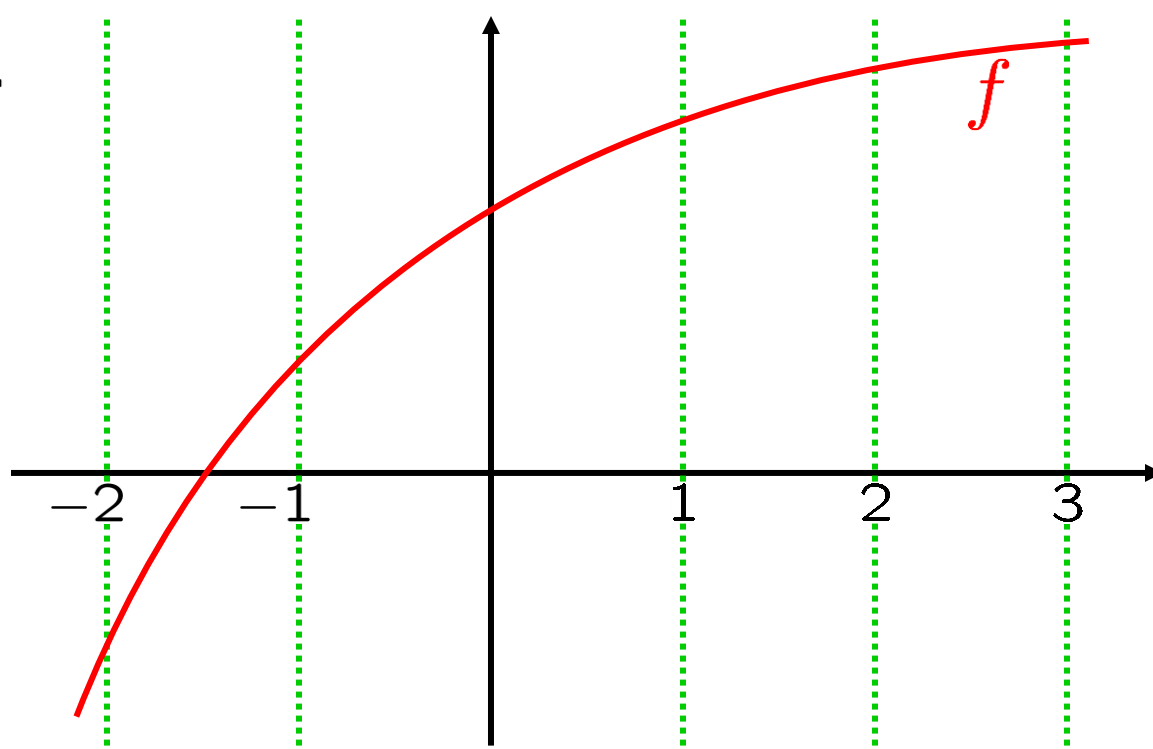
a. Find the average velocity between time $t = 3$ and time $t = 3+h$.

b. Find the instantaneous velocity at time $t = 3$.

0270-4. A heavy object is taken to the top of a building 100 feet high. At time $t = 0$, it is thrown upward at 35 feet/second. We engage the services of two Nobel prize-winning physicists who confer (*i.e.*, yell and scream at one another). After several hours of scholarly study, followed by minor medical treatment for blunt trauma, lacerations and contusions, they hold a joint press conference, and inform their public that, t seconds after release, the object will be located

$$100 + 35t - 16t^2 \quad \text{feet}$$

above the ground. Based on this, **find** the the velocity of the object 0.5 seconds after release. **Give** your answer in feet per second.



Order these numbers, from smallest to largest:

$$f'(-2), f'(-1), f'(0), f'(1), f'(2), f'(3)$$

Note that we are asking about f' , **not** f .



0270-6. Let $f(x) = \frac{2x + 3}{5x - 1}$.

Do NOT use the quotient rule.
Use only the definition of the
derivative as the limit of the
difference quotient.

a. Compute $f'(2)$.

b. Compute $f'(3)$.

c. Compute $f'(4)$.

d. Compute $f'(a)$, for an arbitrary number a .

0270-7. Find a function f and a number a s.t.

OLD

$$f'(a) = \lim_{h \rightarrow 0} \frac{[\cos(2 + h)] - [\cos 2]}{h}.$$