CALCULUS
Linearity of the derivative, and derivatives of polynomials

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0320-1. A car is traveling on a number line,
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on which the unit of distance is a mile. Its position at time t is $(t^3/3) - (5t^2/2) + 6t + 1$, with time measured in hours.

a. What is its velocity at time t, in miles/hr?

b. Graph its velocity, as a function of time.

c. When is its velocity equal to 0?

d. On what (maximal) intervals is the car moving in the positive direction?

e. On what (maximal) intervals is the car moving in the negative direction?

f. On what (maximal) intervals is the car's acceleration positive?

g. On what (maximal) intervals is the car's acceleration negative?

0320-2. A particle is traveling on a number line
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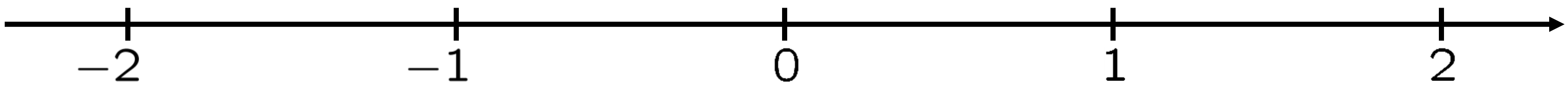
The positive direction is to the right, *viz.*:



The position of the particle, at time t, is $t^2 - 6t + 10$.

a. What is its velocity at time t?

b. When is its velocity equal to 0?

c. On what (maximal) intervals is the particle moving to the left?

d. On what (maximal) intervals is the particle moving to the right?

e. At what time is the particle farthest left?

f. What is its minimial (*i.e.*, leftmost) position?

0320-3. A rock is thrown on the moon.
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Its initial velocity (straight upward) is 15 meters/second.
Its height above the lunar surface, t seconds after release, is

$$h(t) \doteq -(0.82)t^2 + 15t + 2,$$

in meters.

a. What is its velocity at time t, in meters per second?

b. When is its velocity equal to 0?

c. For how long a time (in seconds), after release, is the rock moving upward?

d. What is the maximal height above the lunar surface reached by the rock, in meters?

0320-4. We pump air into a cubical balloon in such a way that its side length at time t is equal to $2t$.

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Its volume is (side length)^3, and its surface area is 6(side length)^2.

a. Find a formula for its volume at time t.

b. Find a formula for the rate of change in its volume at time t.

c. Find a formula for its surface area at time t.

0320-5. We pump air into a spherical balloon
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in such a way that its diameter at time t is equal to $2t$.

Its volume is $\frac{4}{3}\pi(\text{radius})^3$, and its surface area is $4\pi(\text{radius})^2$.

a. Find a formula for its volume at time t.

b. Find a formula for the rate of change in its volume at time t.

c. Find a formula for its surface area at time t.

0320-6. The gravitational force (in newtons)
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exerted by the earth on the moon is given by the formula $F \doteq (2.93 \times 10^{37})/r^2$, where r is their distance apart in km.

a. If the distance increases from 380,000 km to 390,000 km then what is the corresponding change in force (in newtons)?

That is, compute $[F]^{r:\to 390000}_{r:\to 380000}$.

b. Compute the difference quotient $\left([F]^{r:\to 390000}_{r:\to 380000}\right)/10000$.

c. Compute $[dF/dr]_{r:\to 385000}$.

0320-7. The speed of sound (in meters/sec) is

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$c \doteq 20\sqrt{\theta + 273}$, where θ is the air temperature (in Celcius).

a. If the air temperature increases from 0° Celcius to 10° Celcius, then what is the corresponding change in the speed of sound (in meters/second)?

That is, compute $[c]_{\theta:\to 0}^{\theta:\to 10}$.

b. Compute the difference quotient

$$\left([c]_{\theta:\to 0}^{\theta:\to 10}\right)/10.$$

c. Compute $[dc/d\theta]_{\theta:\to 5}$.

0320-8. We study the populations of two species, wolves and sheep, on a certain plot of land.

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Let S be the number of sheep at time t and let W be the number of wolves at time t.

We model the population counts as follows:

$$dW/dt = 5S - 3W - 3000$$
$$dS/dt = 3S - 2W - 1000$$

At what counts, W and S, will the population be stable?

(Stability means: $dW/dt = 0 = dS/dt$.)

0320-9. The position of a particle along a number line is given by

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$$p(t) = (0.03)t^7 - (0.001)t^6 + (0.2)t^5 + 5t^4 + 6t^3 - 2t^2 + t + 8.$$

Compute its velocity, acceleration, jerk, snap, crackle and pop at time t.