CALCULUS
Implicit differentiation
OLD

0430-1.Let an expression y of x be given, implicitly, by the formula $xy - 5x - 8x^3 = 7$.

OLD

a. Find dy/dx by implicit differentiation.

b. Solve for y as an explicit expression of x.

c. Differentiate your answer to Part b, writing dy/dx as an explicit expression of x.

d. Substitute your answer for Part b into every y appearing in your answer to Part a, writing dy/dx as an explicit expression of x.

e. Verify that your answers to Part c and Part d are the same.

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0430-2.Let an expression y of x be given, implicitly, by the formula $x^3 + y^3 = 1$.

OLD

a. Find dy/dx by implicit differentiation.

b. Solve for y as an explicit expression of x.

c. Differentiate your answer to Part b, writing dy/dx as an explicit expression of x.

d. Substitute your answer for Part b into every y appearing in your answer to Part a, writing dy/dx as an explicit expression of x.

e. Verify that your answers to Part c and Part d are the same.

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0430-3. Let an expression y of x be given, implicitly, by the formula
$$xe^y - \cos y + 2e^x \tan y = 3.$$
Find dy/dx by implicit differentiation.

0430-4. Let an expression y of x be given, implicitly, by the formula
$$\sin y = 2x - y.$$
Find dy/dx by implicit differentiation.

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0430-5. Let an expression y of x be given, implicitly, by the formula
$$x^3 + y^3 = 9.$$
Find an equation of the tangent line to the graph of this equation at the point $(1, 2)$.

0430-6. Let an expression y of x be given, implicitly, by the formula
$$y^2 = 3x^4 - 2x^2.$$
Find an equation of the tangent line to the graph of this equation at the point $(1, 1)$.

. OLD Let an expression y of x be given, implicitly, by the formula
$$3x^5 - y^5 = 8 + xy.$$
Find d^2y/dx^2 by implicit differentiation.

. OLD Let an expression y of x be given, implicitly, by the formula
$$\sqrt{3}x + y^7 = 4 + xy.$$
Find d^2y/dx^2 by implicit differentiation.

For every $a \in \mathbb{R}$, for every $b > 0$,

OLD

let G_a be graph of the equation $y = ax^5$ and
let H_b be graph of the equation $x^2 + 5y^2 = b$.

a. Let p be the point $(1, 1)$, which lies
      both on G_1 and on H_6.

   Show that the tangent lines to G_1 and H_6
      at p are perpendicular.

b. Let a and b be any two real numbers,
      with $b > 0$.

   Let q be any point which lies
      both on G_a and on H_b.

   Show that the tangent lines to G_a and H_b
      at q are perpendicular.

Challenge problem (not assigned):

For every $a, b \in \mathbb{R}$,

let G_a be graph of $x - 2y = 2axy$ and
let H_b be graph of $x^3 + 2y^3 = b$.

a. Let p be the point $(2, 1)$, which lies
both on G_0 and on H_{10}.
Show that the tangent lines to G_0 and H_{10}
at p are perpendicular.

b. Let a and b be any two real numbers.
Let q be any point which lies
both on G_a and on H_b.
Show that the tangent lines to G_a and H_b
at q are perpendicular.