CALCULUS
The Mean Value Theorem

OLD

0460-1. Let $f(x) = 2 - 6x + x^2$.
OLD

a. Check that f satisfies the conditions of Rolle's Theorem on the interval $[2, 4]$. That is, check

(i) that f is continuous on $[2, 4]$,
(ii) that f is differentiable on $(2, 4)$
and (iii) that $f(2) = f(4)$.

b. Find all solutions to the equation in the conclusion of Rolle's Th'm for f on $[2, 4]$. That is, find all $c \in (2, 4)$ s.t. $f'(c) = 0$.

0460-2. Let $f(x) = 2 - 3x + x^2$.
OLD

a. Check that f satisfies the conditions of the MVT on the interval $[2, 4]$.
That is, check

(i) that f is continuous on $[2, 4]$

and (ii) that f is differentiable on $(2, 4)$.

b. Find all solutions to the equation in the conclusion of the MVT for f on $[2, 4]$.
That is, find all $c \in (2, 4)$ s.t.

$$f'(c) = \frac{[f(4)] - [f(2)]}{4 - 2}.$$

0460-3. Let $f(x) = 8 + |x - 3|$.
OLD

a. Show that f is continuous on $[1, 5]$.

b. Show that $f(1) = f(5)$.

c. Show that the conclusion of Rolle's Th'm, for f on $[1, 5]$, fails. That is, show that there is no $c \in (1, 5)$ s.t. $f'(c) = 0$.

d. Explain why this does not contradict Rolle's Theorem.

0460-4. Let $f(x) = 2x + |x - 3|$.
OLD

a. Show that f is continuous on $[1, 5]$.

b. Show that the conclusion of the MVT, for f on $[1, 5]$, fails. That is, show that there is no $c \in (1, 5)$ s.t.

$$f'(c) = \frac{[f(5)] - [f(1)]}{5 - 1}.$$

c. Explain why this does not contradict the MVT.

0460-5. Let $f(x) = \begin{cases} 21, & \text{if } x = 1 \\ x^2, & \text{if } 1 < x < 5 \\ 25, & \text{if } x = 5. \end{cases}$

OLD

a. Show that f is differentiable on $(1,5)$.

b. Show that the conclusion of the MVT, for f on $[1,5]$, fails. That is, show that there is no $c \in (1,5)$ s.t.

$$f'(c) = \frac{[f(5)] - [f(1)]}{5-1}.$$

c. Explain why this does not contradict the MVT.

0460-6. Show that $3x + 5 - \cos x = 0$ has exactly one real solution.
OLD

0460-7. Let c be any constant.
OLD

Show that $x^3 - 12x + c = 0$ has at most one real solution on $[3, 4]$.

0460-8. OLD At noon on some day, a certain car is at the 100 mile marker on some road. The speed limit on the road is 30 mph. A driver drives the car for four hours, obeying the speed limit.

Let $f(t)$ denote the position of the car t hours after noon; then

$$f(0) = 100$$

and

$$\forall t \in [0,4], \quad f'(t) \leq 30.$$

With these constraints, what is the largest possible value for $f(4)$?