

CALCULUS

Definite integration and Riemann sum problems

OLD

0590-1. Let $f(x) = 4 - x^2$.

OLD

a. Compute $L_4S_{-2}^2f$.

Sketch f over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $L_4S_{-2}^2f$.

b. Compute $M_4S_{-2}^2f$.

Sketch f over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $M_4S_{-2}^2f$.

c. Compute $R_4S_{-2}^2f$.

Sketch f over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $R_4S_{-2}^2f$.

0590-2. Let $f(x) = e^x + 5$.

OLD

- Compute $L_3 S_0^6 f$ to three decimal places.
- Compute $M_3 S_0^6 f$ to three decimal places.
- Compute $R_3 S_0^6 f$ to three decimal places.

0590-3. Let $f(x) = \sin^2 x$.

OLD

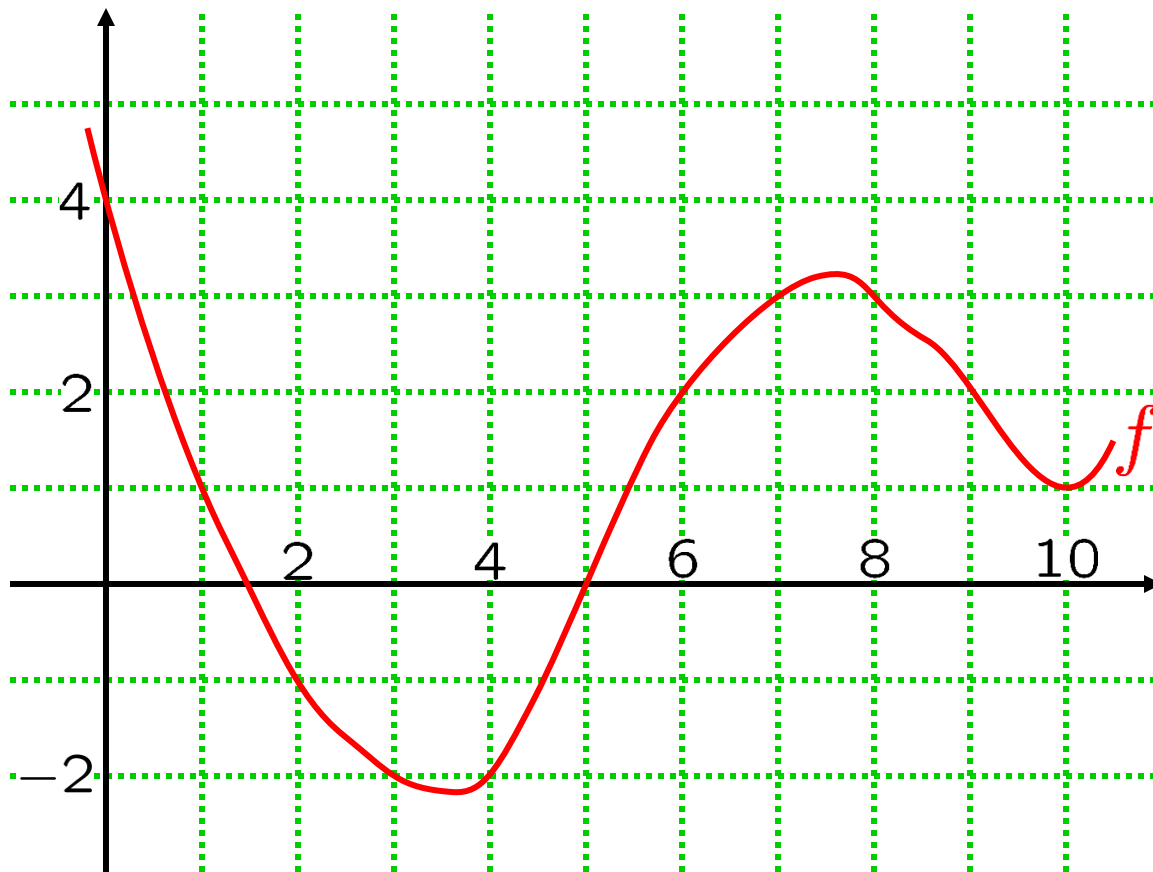
- Compute $L_5 S_0^\pi f$ to three decimal places.
- Compute $M_5 S_0^\pi f$ to three decimal places.
- Compute $R_5 S_0^\pi f$ to three decimal places.

^{OLD} 0590-4. A car's acceleration is positive from time 0 to time 12 seconds, and its velocity at various times is given in the table below.

time (secs)	0	2	4	6	8	10	12
velocity (ft/sec)	0	20	35	45	53	59	62

Find upper and lower estimates for the distance traveled by the car over these 12 seconds.

0590-5. The graph of a function f appears below.
OLD



Estimate $\int_0^{10} f(x) dx$ by computing

(a) $L_5 S_0^{10} f$, (b) $M_5 S_0^{10} f$

and (c) $R_5 S_0^{10} f$.

0590-6. Express the area under $y = e^{-x^2}$ from $x = -1$ to $x = 1$ as a limit of midpoint Riemann sums. (Don't evaluate the limit.)

0590-7. Express the area under $y = \sqrt{x^3 + x + 5}$ from $x = 2$ to $x = 4$ as a limit of left endpoint Riemann sums. (Don't evaluate the limit.)

0590-8. Express the area under $y = \cos(x^3)$ from $x = -2$ to $x = 5$ as a limit of right endpoint Riemann sums. (Don't evaluate the limit.)

0590-9. Express $\int_5^{13} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx$ as a limit of midpoint Riemann sums.
(Don't evaluate the limit.)

0590-10. Let $f(x) = x^3$.

a. Write $R_n S_0^2 f$ as a rational expression in n (i.e., as one polynomial in n divided by another).

b. Compute $\lim_{n \rightarrow \infty} R_n S_0^2 f$.

0590-11. The limit
OLD

$$\lim_{n \rightarrow \infty} \left[\frac{4}{n} \sum_{j=1}^n \left(e^{\sin(3 + (4/n)j)} \right) \right]$$

represents the area under $y = f(x)$
from $x = a$ to $x = b$,
for some choice of $f(x)$, a and b .

- a. Find $f(x)$, a and b .
- b. Express the limit as a definite integral.

0590-12. The limit
OLD

$$\lim_{n \rightarrow \infty} \left[\frac{7}{n} \sum_{j=0}^{n-1} \left(\cos \left(e^{2+(7/n)j} \right) \right) \right]$$

represents the area under $y = f(x)$
from $x = a$ to $x = b$,
for some choice of $f(x)$, a and b .

- a. Find $f(x)$, a and b .
- b. Express the limit as a definite integral.

0590-13. Let $f(x) = 3 + \sqrt{4 - x^2}$.

OLD

a. Sketch the graph of $y = f(x)$.

b. Compute $\int_{-2}^2 f(x) dx$, by interpreting this integral as an area.