CALCULUS
The Fundamental Theorems of Calculus, problems

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0620-1. The graph of g is shown below.
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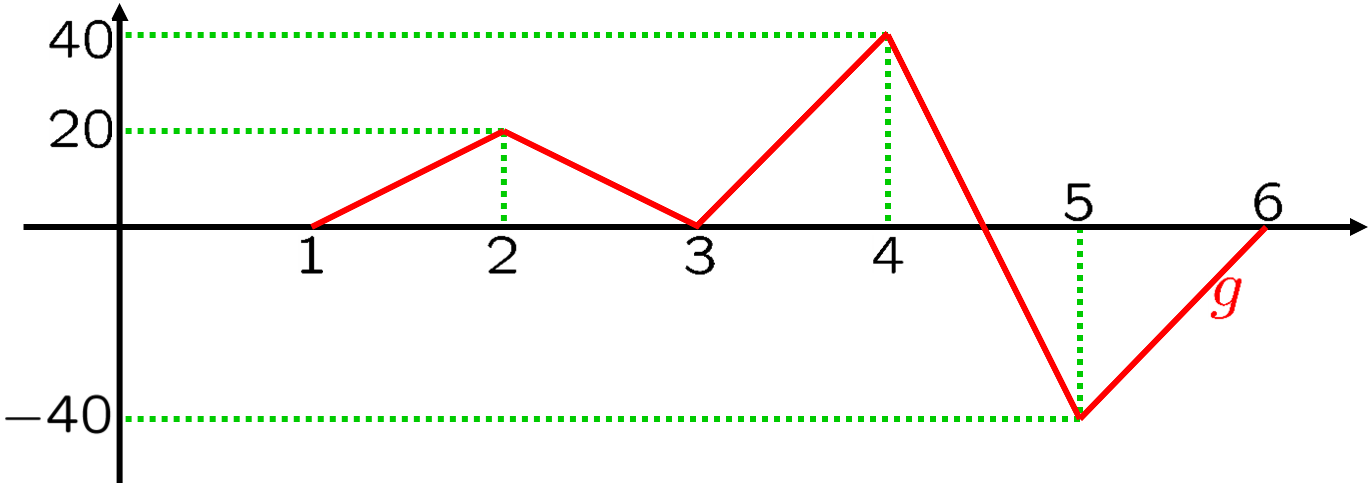
Let $f(x) = \int_1^x g(t)\,dt$.



a. Compute $f(6)$.

b. Find the maximal intervals of increase and decrease for f.

c. At what numbers does f have a local max and local min?

d. Find the maximal intervals of concavity for f.

e. What are the points of inflection for f?

0620-2. Let $f(x) = \int_2^x t^3 + t\,dt$.
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a. Compute a (polynomial) formula for $f(x)$.

b. Compute a (polynomial) formula for $f'(x)$.

0620-3. Let $f(x) = \int_0^x e^{-t^2}\,dt$.
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a. Sketch $y = e^{-t^2}$, then choose some number on the t-axis, label it as x, and shade in a region under the graph whose area is $f(x)$.

b. Compute a formula for $f'(x)$.

0620-4. Compute $\dfrac{d}{dx}\displaystyle\int_{3}^{x} \cos(t^5)\,dt$.
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0620-5. Compute $\dfrac{d}{dx}\displaystyle\int_{x}^{3} \cos(t^5)\,dt$.
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0620-6. Compute $\dfrac{d}{dx}\displaystyle\int_{3}^{x^2} \cos(t^5)\,dt$.
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0620-7. Compute $\dfrac{d}{dx}\displaystyle\int_{3}^{x^3} \cos(t^5)\,dt$.
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0620-8. Compute $\dfrac{d}{dx}\displaystyle\int_{x^2}^{3} \cos(t^5)\,dt$.
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0620-9. Compute $\dfrac{d}{dx}\displaystyle\int_{x^2}^{x^3} \cos(t^5)\,dt$.
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0620-10. Compute $\frac{d}{dq}\int_{\cos q}^{5+q^4} e^{-s^4+3s}\,ds$.
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0620-11. Compute $\frac{d}{dt}\int_{4}^{5+t^2} \arctan u\,du$.
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0620-12. Compute $\frac{d}{ds}\int_{-s^4}^{0} \ln(3+v^2)\,dv$.
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0620-13. Evaluate $\int_2^3 \left(x^2 + \frac{1}{x^2}\right)\, dx$.
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0620-14. Evaluate $\int_1^6 \frac{7x^3 - 2x^2 - 4x + 8}{x}\, dx$.
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0620-15. Evaluate $\int_{\pi/4}^{\pi/3} \cos t\, dt$.
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0620-16. Evaluate $\int_{\pi/4}^{\pi/3} (\sec t)(\tan t)\, dt$.
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0620-17. Evaluate $\int_0^1 \frac{1}{1+u^2}\, du$.
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0620-18. Evaluate $\int_{-3}^{4} (2x - 2|x|)\, dx$.
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0620-19. Evaluate $\lim_{n\to\infty} \frac{1}{n}\left[\sum_{j=0}^{n-1} (j/n)^2\right]$.
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by converting to a definite integral, and then using the Fundamental Theorem of Calculus.

0620-20. Evaluate $\lim_{n\to\infty} \frac{9}{n}\left[\sum_{j=0}^{n-1} (9j/n)^7\right]$.
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by converting to a definite integral, and then using the Fundamental Theorem of Calculus.

0620-21. Evaluate $\lim_{n\to\infty} \frac{\pi}{n}\left[\sum_{j=1}^{n} \sin(\pi j/n)\right]$.
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by converting to a definite integral, and then using the Fundamental Theorem of Calculus.

0620-22. Water starts pouring from a tank.
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After t minutes, the rate of flow, out of the tank is $1 + 3t^2$ gallons per minute. How many gallons pour out between 2 and 3 minutes after the start?

0620-23. A model rocket is launched and starts
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climbing. After t seconds, its altitude is increasing at $1 + 3t^2$ feet/second. How much does its altitude increase between 2 and 3 seconds after launch?

0620-24. At x ounces, the marginal cost of
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production for certain liquid is $1 + 3x^2$ dollars per ounce. How much does it cost to increase production from 2 to 3 ounces?

0620-25. A rope lies along a number line,
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between 0 and 100. The weight density of the rope at x is $1 + 3x^2$ pounds per inch. How much does the portion of the rope $x = 2$ and $x = 3$ weigh?

0620-26. By definition, if a force of F is
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applied to a particle over a distance s, then the **work** done is Fs. A 10 foot rope hangs from the top of a wall, and its density is 2 ounces per foot. We pull the rope up over the wall. Each particle of rope is acted on by a force equal to its weight, until it reaches the top of the wall (after which it simply coils up on the roof, which involves no work). How much work is done in pulling the rope up?

0620-27. By definition, if a force of F is applied to a particle over a distance s, then the **work** done is Fs. A certain object is lying on a frictionless horizontal number line, attached to a horizontal spring, which, in turn, is attached to a vertical wall. The wall crosses the number line at -1, and the object is positioned on the number line at 0. We pull the object from 0 to 4. Assume that the spring pulls back with a force of $2x$, when the object is positioned at x. Compute the total work done by the spring.

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