CALCULUS
Volume by slices and the disk and washer methods: Problems

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0720-1. Let R be the region bounded by

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$y = x + 1$ and $x = 2$ in $1 \leq y \leq 2$.

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

c. Find the volume of the solid obtained by rotating R about the y-axis.

0720-2. Let R be the region bounded by

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$y - 1 = (x - 1)^2$ and $y = x$.

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

c. Find the volume of the solid obtained by rotating R about the y-axis.

0720-3. OLD Let R be the region bounded by

$$y = \ln x,\ x = 4 \text{ and } y = 1.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the y-axis.

0720-4. OLD Let R be the region bounded by

$$y = \sin x \text{ and } y = 0 \text{ in } 0 \leq x \leq \pi.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

Hint: $\sin^2 x = \dfrac{1 - [\cos(2x)]}{2}$

0720-5. Let R be the region bounded by
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$$x^2 + (y-3)^2 = 1.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

Note: This solid is called a torus. It is in the shape of a doughnut.

Hint: Remember that $2\int_{-1}^{1} \sqrt{1-x^2}\,dx$ is known; it is the area enclosed in a circle of radius 1.

0720-6. Let R be the region bounded by
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$$y = x^3 \text{ and } x = y^4.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the line $y = -1/2$.

c. Find the volume of the solid obtained by rotating R about the line $x = -1/3$.

0720-7. Let R be the region bounded by
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$$y = x^2 \text{ and } x = y^6.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the line $y = -1/2$.

c. Find the volume of the solid obtained by rotating R about the line $x = -1/3$.

0720-8. Let R be the region bounded by
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$$y = 4\cos x,\ y = e^x \text{ in } 0 \le x \le \pi/4.$$

Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating R about the line $y = 5$.

0720-9. Describe the solid of revolution
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whose volume is given by

$$\pi \int_1^2 \left(9e^{8x} - 4e^{2x}\right)\, dx.$$

Do not evaluate this integral.

0720-10. Describe the solid of revolution
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whose volume is given by

$$\pi \int_{\pi/2}^{\pi} (2 + \sin x)^2 - 4\, dx.$$

Do not evaluate this integral.

0720-11. A solid S sits above a horizontal plane P. $\forall x \geq 0$, let P_x be the horizontal plane that is x units above P. Suppose that S lies between P_1 and P_2. Suppose, also, that $\forall x \in [1, 2]$, the intersection of S and P_x is the region inside an ellipse

whose major axis has length x
and whose minor axis has length e^{2x^2}.

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Compute the volume of S.

Hint: Remember that if a and b are the major and minor axes of an ellipse E, then the area inside E is $\pi ab/4$.

0720-12. Using the disk method, find the volume in a ball of radius 5, following the diagram shown below.

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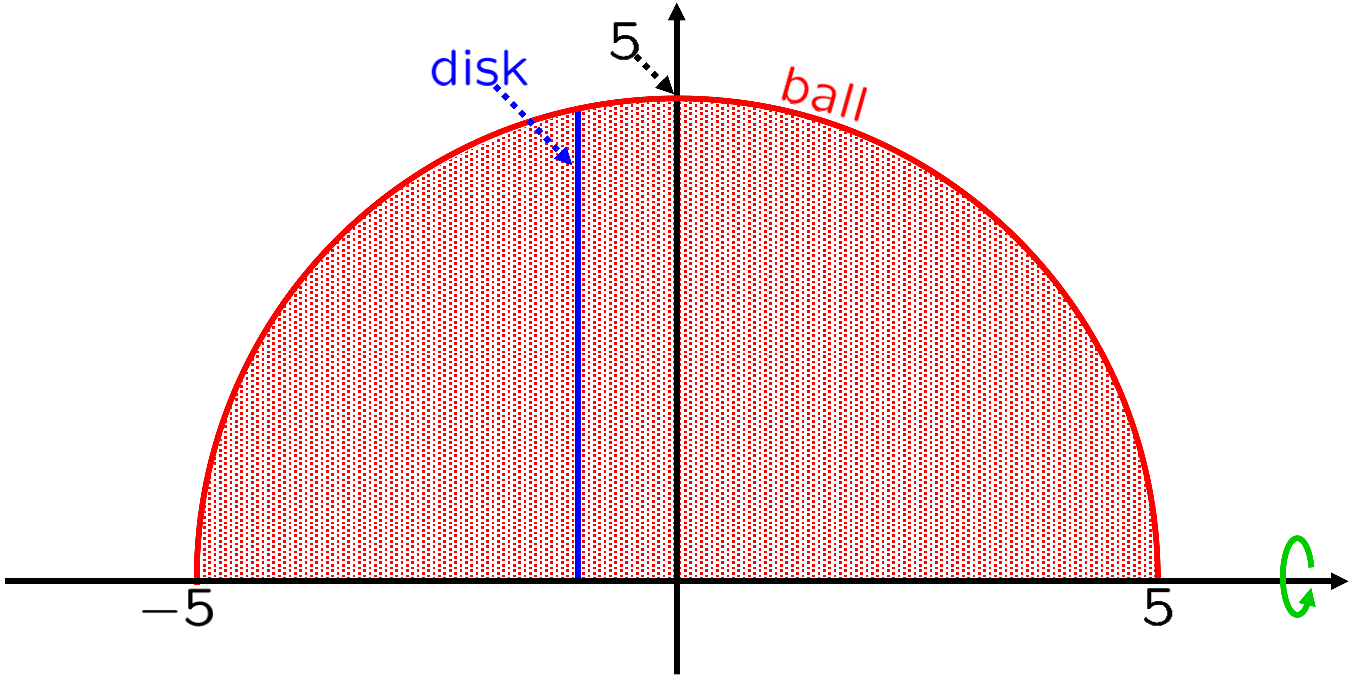

0720-13. We create a napkin holder by drilling
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a cylindrical hole of radius 4 through the middle of a ball of radius 5, as shown below. Using the washer method, find its volume.