Math 1271 Section 30 Recitation Quiz 3

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Time limit: 20 minutes | TA: ______

NO CALCULATORS. NO HANDHELD DEVICES. NO BOOKS OR REFERENCE MATERIALS OF ANY KIND.

1. (40 points) Consider the function

$$f(x) = \frac{1}{(2+x)^2}.$$

By using the definition of the derivative, find $f'(2)$. You are not allowed to use the differentiation rules.

$$\frac{(f(2+h))-(f(2))}{h} = \frac{1}{h}\left[\frac{1}{(4+h)^2} - \frac{1}{4^2}\right] = \frac{1}{h}\left[\frac{4^2-(4+h)^2}{(4+h)^2\cdot 4^2}\right]$$

$$= \frac{1}{h}\left[\frac{4^2-(4^2+8h+h^2)}{(4+h)^2\cdot 4^2}\right] = \frac{1}{h}\left[\frac{-8h-h^2}{(4+h)^2\cdot 4^2}\right]$$

$$= \frac{-8-h}{(4+h)^2\cdot 4^2} \xrightarrow{h\to 0} \frac{-8}{4^2\cdot 4^2} = -\frac{8}{16\cdot 16} = -\frac{1}{2\cdot 16}$$

$$= -\frac{1}{32}$$

2. (15 points) Is the following statement true or false?

$$\lim_{x\to-\infty} \ln\left(\frac{-1}{x}\right) = -\infty$$

True

False

$$\boxed{\text{“}\frac{-1}{-\infty} = 0^+\text{”} \quad \Big| \quad \text{“}\ln(0^+) = -\infty\text{”}}$$

SEE OTHER SIDE FOR MORE PROBLEMS.

3. (25 points) Find

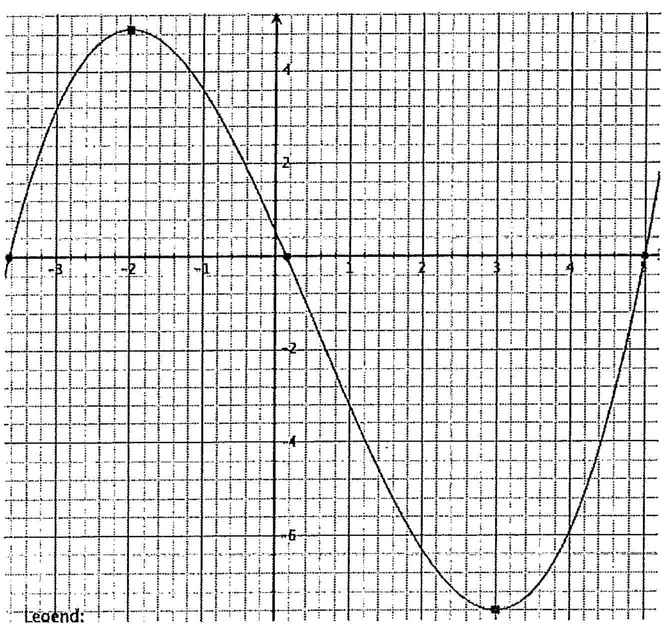
$$\lim_{x\to\infty} \frac{4x^2+7x-8x^3+19}{3x^2+6x+5}.$$

asymptotics

$$\lim_{x\to\infty} \frac{-8x^3}{3x^2} = \lim_{x\to\infty} -\frac{8}{3}x = -\infty$$

$$\boxed{\text{"}\; -\frac{8}{3}\cdot\infty = -\infty \;\text{"}}$$

4. (20 points) Suppose $f(x)$ has the following graph.



Which of the following is true about the derivative function $f'(x)$?

(A) $f'(-2) > 0$. ✗   0

(B) $f'(-1) < 0$. ✓   neg

(C) $f'(3) < 0$. ✗   0

(D) $f'(5) = 0$. ✗   pos

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