Math 1271 Quiz 7

March 27, 2014

Name: SOLUTIONS

TA: _____

NO CALCULATORS. NO HANDHELD DEVICES. NO BOOKS OR REFERENCE MATERIALS OF ANY KIND.
Time allowed: 20 minutes; Grader: Ashley Earls. Good luck!

1. Let $f(x) = 2x^2 - 3x + 1$.

   (a) (*20 points*) Verify that f satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2]$.

f is a polynomial, so

f is continuous on $[0,2]$

and f is differentiable on $(0,2)$.

   (b) (*15 points*) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f'(x) = 4x - 3$$

$$4c - 3 = f'(c) = \frac{(f(2)) - (f(0))}{2 - 0} = \frac{8 - 6 + 1 - 1}{2} = \frac{2}{2} = 1$$

$$4c = 4$$

$$c = 1 \in (0, 2)$$

$c = 1$ is the only answer

2. (*15 points, no partial credit*) Let $f(x) = \tan(x)$. Note that $f(0) = f(\pi) = 0$.

   True or false? By Rolle's Theorem, there exists $c \in (0, \pi)$ with $f'(c) = 0$.

   True

   (False)

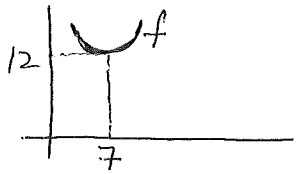
$f'(c) = \sec^2 c = \frac{1}{\cos^2 c}$ is never 0.

3. (*15 points, no partial credit*) Let $f : \mathbb{R} \to \mathbb{R}$ be any continuous function.

True or false? If $f(7) = 12$, $f'(7) = 0$, and $f''(7) = 3$, then f has a local minimum at $x = 7$.

(True)

False



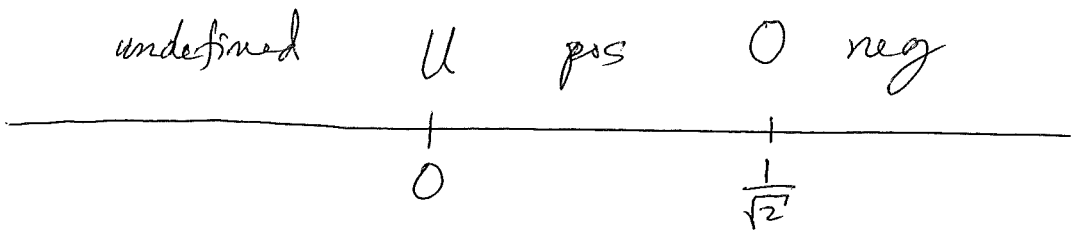
4. (*35 points*) Let $f(x) = x - x^2 - \ln(x)$. Find the maximal intervals of concavity and the inflection points for f.

For all $x \leq 0$, $f(x)$ is undefined, so $f'(x)$ and $f''(x)$ are also undefined.

$$f'(x) \overset{x>0}{=\!=} 1 - 2x - \frac{1}{x} = 1 - 2x - x^{-1}$$

$$f''(x) \overset{x>0}{=\!=} -2 + x^{-2} = \frac{-2x^2+1}{x^2} = \frac{-2\left(x+\frac{1}{\sqrt{2}}\right)\left(x-\frac{1}{\sqrt{2}}\right)}{x^2}$$

f''



f is concave up on $\left(0, \frac{1}{\sqrt{2}}\right]$.

f is concave down on $\left[\frac{1}{\sqrt{2}}, \infty\right)$.

The only point of inflection for f is

$$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} - \frac{1}{2} - \ln\left(\frac{1}{\sqrt{2}}\right)\right).$$

PLEASE SEE THE OTHER SIDE FOR MORE PROBLEMS.