Math 1271 Quiz 8

April, 3, 2014
Name: _SOLUTIONS_
TA: _____

NO CALCULATORS. NO HANDHELD DEVICES. NO BOOKS OR REFERENCE MATERIALS OF ANY KIND.
Time allowed: 20 minutes; Grader : Amit Sharma. Good luck!

1. (*35 points*) Find the points on the ellipse
$$4x^2 + y^2 = 4 \implies y^2 = 4 - 4x^2$$
that are farthest away from the point $(1, 0)$.

Maximize $(x-1)^2 + y^2 = \underbrace{(x-1)^2 + 4 - 4x^2}_{= f(x)}$

$f'(x) = 2(x-1) - 8x$
$\quad = -6x - 1$

pos   0   neg

$x \qquad\qquad -\frac{1}{6}$

$f(x)$ attains its global max at $x = -\frac{1}{6}$

$y^2 = 4 - 4x^2 = 4 - \frac{4}{36} = 4 - \frac{1}{9} = \frac{36}{9} - \frac{1}{9} = \frac{35}{9}$

$y = \pm \frac{\sqrt{35}}{3}$

Two points = $\left(-\frac{1}{6}, -\frac{\sqrt{35}}{3}\right), \left(-\frac{1}{6}, \frac{\sqrt{35}}{3}\right)$

2. (*15 points*) State whether the following statement is true or false:
Let a function f be differentiable at x_n, and suppose that $f'(x_n) \neq 0$. Let L be the tangent line to the graph of f at $(x_n, f(x_n))$. Let x_{n+1} be the x-intercept of L. Then
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$
(True)   (l'Hospital's Rule)

PLEASE SEE THE OTHER SIDE FOR MORE PROBLEMS.

3. (*15 points*) State whether the following statement is true or false:
   The line $x = 0$ is NOT a vertical asymptote of the function $f(x) = \frac{1}{x}$ because the point $x = 0$ is not in the domain of the function.

   (False)

4. (*35 points*) Approximate the number $\sqrt{3.98}$ by finding a linearization of the function $f(x) = \sqrt{x+4}$ at the point $(0, 2)$. Show all your work.

$$f(x) = \sqrt{x+4} \qquad\qquad f(0) = \sqrt{4} = 2$$

$$f'(x) = \frac{1}{2\sqrt{x+4}} \qquad\qquad f'(0) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$\underline{\text{Linearization}:} \quad L(x) = 2 + \frac{1}{4}(x-0) = 2 + \frac{x}{4}$$

$$\sqrt{3.98} = f(-0.02) \approx L(-0.02) = 2 + \frac{-0.02}{4}$$

$$= 2 - 0.005$$

$$= 1.995$$

PLEASE SEE THE OTHER SIDE FOR MORE PROBLEMS.