Math 1271 Quiz 10

April 24, 2014

Name: SOLUTIONS TA: _____

NO CALCULATORS. NO HANDHELD DEVICES. NO BOOKS OR REFERENCE MATERIALS OF ANY KIND.
Time allowed: 20 minutes; Grader: Ashley Earls. Good luck!

1. Compute the following integral:

$$\int_0^{\sqrt{2}} \frac{x}{\sqrt{1+4x^2}}\,dx$$

$$= \int_1^9 \frac{\frac{1}{8}\,du}{\sqrt{u}}$$

$$= \frac{1}{8}\int_1^9 u^{-1/2}\,du$$

$$= \frac{1}{8}\left[\frac{u^{1/2}}{1/2}\right]_{u=1}^{u=9}$$

$$= \frac{1}{8}\left[\frac{9^{1/2}}{1/2} - \frac{1^{1/2}}{1/2}\right]$$

$$= \frac{1}{8}\left[\frac{3}{1/2} - \frac{1}{1/2}\right]$$

$$= \frac{1}{8}[6-2] = \frac{4}{8} = \frac{1}{2}$$

$u = 1 + 4x^2$

x	u
$\sqrt{2}$	9
0	1

$du = 8x\,dx$

$\frac{1}{8}du = x\,dx$

2. (*15 points, no partial credit*) True or false? The average value of the function $f(x) = 4x - x^2$ on the interval $[2,5]$ is

$$\frac{1}{3}\int_2^5 4x - x^2\,dx$$

(True)

False

3. (*15 points, no partial credit*) Let $f(x)$ and $g(x)$ be any functions. True or false?

$$\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$$

True

(False)

4. (*35 points*) Find the area enclosed by the line $y = x + 1$ and the parabola $y = \frac{1}{2}x^2 - 3$. Note that the only two points of intersection for these graphs are $(-2, -1)$ and $(4, 5)$. No need to work out the arithmetic.

The parabola is concave up, so it lies below all of its secant line segments

$\frac{1}{2}x^2 - 3 < x + 1$ on $-2 < x < 4$

$$\text{Area enclosed} = \int_{-2}^{4} \left[(x+1) - \left(\tfrac{1}{2}x^2 - 3\right)\right] dx$$

$$= \int_{-2}^{4} \left[-\tfrac{1}{2}x^2 + x + 4\right] dx$$

$$= \left[-\frac{1}{2}\cdot\frac{x^3}{3} + \frac{x^2}{2} + 4x\right]_{x:\,-2}^{x:\,4}$$

$$= \left[-\frac{1}{2}\cdot\frac{4^3}{3} + \frac{4^2}{2} + 4\cdot 4\right] - \left[\frac{1}{2}\cdot\frac{8}{3} + \frac{4}{2} - 8\right]$$

PLEASE SEE THE OTHER SIDE FOR MORE PROBLEMS.