CALCULUS
The Sigma notation

cf. §7.2, p. 144 sigma notation:

$$\sum_{j=p}^{q} a_j := a_p + a_{p+1} + \cdots + a_q$$

e.g.:

$$\sum_{j=1}^{7} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2$$

$$\sum_{j=1}^{n} j^2 = 1^2 + 2^2 + \cdots + n^2$$

$$\sum_{j=1}^{n} j^2 = 1^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

cf. §7.2, p. 144 sigma notation:

$$\sum_{j=p}^{q} a_j := a_p + a_{p+1} + \cdots + a_q$$

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{j=1}^{n} j^3 = \frac{n^2(n+1)^2}{4}$$

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$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{j=1}^{n} j^3 = \frac{n^2(n+1)^2}{4} = \left(\sum_{j=1}^{n} j\right)^2$$

cf. §7.2, p. 144 sigma notation:

$$\boxed{\sum_{j=p}^{q} a_j} := a_p + a_{p+1} + \cdots + a_q$$

The "linear operations" are: addition and scalar multiplication.

$\displaystyle\sum_{j=p}^{q}$ is linear, i.e., both of the following conditions hold:

FIRST, $\sum$ distributes over addition, *i.e.*, $\sum$ is additive:

$$\sum_{j=p}^{q} (a_j + b_j) = \left(\sum_{j=p}^{q} a_j \right) + \left(\sum_{j=p}^{q} b_j \right),$$

i.e., $(a_p + b_p) + \cdots + (a_q + b_q)$

$$= (a_p + \cdots + a_q) + (b_p + \cdots + b_q)$$

SECOND, $\sum$ "commutes" with scalar multiplication:

"Commutes" refers to traveling. c and $\sum$ travel past one another.

$$\sum_{j=p}^{q} c a_j = c \left(\sum_{j=p}^{q} a_j \right),$$

i.e., $(ca_p + \cdots + ca_q) = c(a_p + \cdots + a_q)$

7.2

cf. §7.2, p. 144 sigma notation:

$$\boxed{\sum_{j=p}^{q} a_j} := a_p + a_{p+1} + \cdots + a_q$$

NOTE: If an expression inside $\sum$ does not depend on the index of summation, then it can be brought outside the $\sum$. For example, ...

common factor

$$\sum_{j=3}^{5} [(k+7)e^j] = [(\boxed{k+7})e^3] + [(\boxed{k+7})e^4] + [(\boxed{k+7})e^5]$$

$$= [\boxed{k+7}][e^3 + e^4 + e^5]$$

$$= [k+7]\left[\sum_{j=3}^{5} e^j\right]$$

$\sum$ "commutes" with scalar multiplication:

$$\sum_{j=p}^{q} c a_j = c\left(\sum_{j=p}^{q} a_j\right),$$

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$$\sum_{j=p}^{q} a_j := a_p + a_{p+1} + \cdots + a_q$$

NOTE: If an expression inside $\sum$ does not depend on the index of summation, then it can be brought outside the $\sum$. For example, . . .

$$\sum_{j=3}^{5} [(k+7)e^j] = [k+7]\left[\sum_{j=3}^{5} e^j\right]$$

does not depend on j

$$= [k+7]\left[\sum_{j=3}^{5} e^j\right]$$

$\sum$ "commutes" with scalar multiplication:

$$\sum_{j=p}^{q} ca_j = c\left(\sum_{j=p}^{q} a_j\right),$$

i.e., $(ca_p + \cdots + ca_q) = c(a_p + \cdots + a_q)$

cf. §7.2, p. 144 sigma notation:

$$\sum_{j=p}^{q} a_j := a_p + a_{p+1} + \cdots + a_q$$

NOTE: If an expression inside $\sum$ does not depend on the index of summation, then it can be brought outside the $\sum$. For example, . . .

$$\sum_{j=3}^{5} [(k+7)e^j] = [k+7]\left[\sum_{j=3}^{5} e^j\right]$$

$$\sum_{j=3}^{5} [(2j-1)e^j] = [5e^3] + [7e^4] + [9e^5]$$

$(2j-1)$ depends on j

no common factor

$\sum$ "commutes" with scalar multiplication:

$$\sum_{j=p}^{q} ca_j = c\left(\sum_{j=p}^{q} a_j\right),$$

i.e., $(ca_p + \cdots + ca_q) = c(a_p + \cdots + a_q)$

cf. §7.2, p. 144 sigma notation:

$$\boxed{\sum_{j=p}^{q} a_j} := a_p + a_{p+1} + \cdots + a_q$$

NOTE: It is important that k be independent of j.
If, for example, $k = 2j - 8$, then . . .

$$\sum_{j=3}^{5} [(k+7)e^j] = [k+7]\left[\sum_{j=3}^{5} e^j\right]$$

$$\sum_{j=3}^{5} [(2j-1)e^j] = [5e^3] + [7e^4] + [9e^5]$$

$\sum$ "commutes" with scalar multiplication:

$$\sum_{j=p}^{q} ca_j = c\left(\sum_{j=p}^{q} a_j\right),$$

i.e., $(ca_p + \cdots + ca_q) = c(a_p + \cdots + a_q)$

cf. §7.2, p. 144 sigma notation:

$$\sum_{j=p}^{q} a_j := a_p + a_{p+1} + \cdots + a_q$$

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||

$$\sum_{j=3}^{5} [(2j-1)e^j] = [5e^3] + [7e^4] + [9e^5]$$

The moral: It's important to track dependencies.

$\sum$ "commutes" with scalar multiplication:

$$\sum_{j=p}^{q} ca_j = c\left(\sum_{j=p}^{q} a_j\right),$$

i.e., $(ca_p + \cdots + ca_q) = c(a_p + \cdots + a_q)$

cf. §7.2, p. 144 sigma notation:

$$\boxed{\sum_{j=p}^{q} a_j} := a_p + a_{p+1} + \cdots + a_q$$

WARNING: $\displaystyle\sum_{j=p}^{q}$ is not multiplicative: Typically,

$$\sum_{j=p}^{q} (a_j b_j) \neq \left(\sum_{j=p}^{q} a_j \right) \left(\sum_{j=p}^{q} b_j \right),$$

i.e., $a_p b_p + \cdots + a_q b_q$

$$\neq (a_p + \cdots + a_q)(b_p + \cdots + b_q),$$

e.g., $(1{\cdot}2)+(3{\cdot}4)+(5{\cdot}6) \neq (1+3+5)(2+4+6)$

Can you find a summation by parts formula?

Next: adding a constant to the index . . .

cf. §7.2, p. 144 sigma notation:

$$\boxed{\sum_{j=p}^{q} a_j} := a_p + a_{p+1} + \cdots + a_q$$

$$j :\to j+3$$

$$\sum_{j=p}^{q} a_{j+3} = a_{p+3} + a_{p+4} + \cdots + a_{q+3}$$

$$p :\to p-3, \; q :\to q-3$$

$$\sum_{j=p-3}^{q-3} a_{j+3} = a_p + a_{p+1} + \cdots + a_q = \sum_{j=p}^{q} a_j$$

The point:

If you add a number to the index variable
and subtract it from the limits of summation,
then the sum stays the same.

STOP

Next: adding a constant to the index . . .