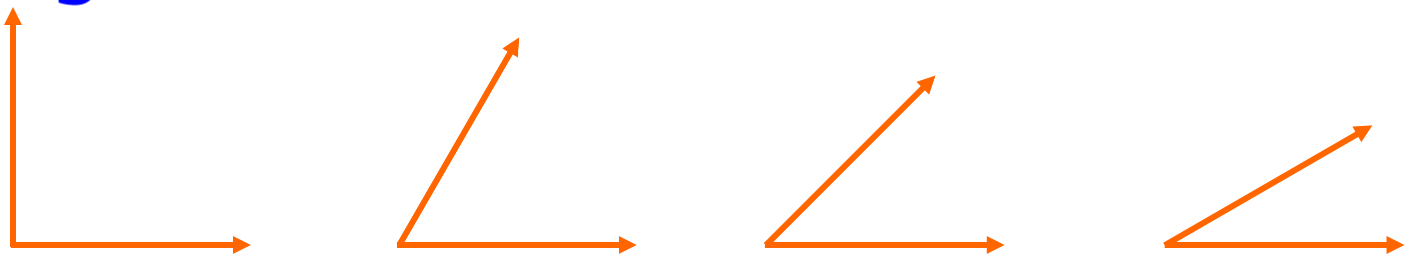
CALCULUS
Basics of trigonometry

What are *DEGREE*s and *RADIAN*s?

A **degree** is one 360^{th} of a full revolution.

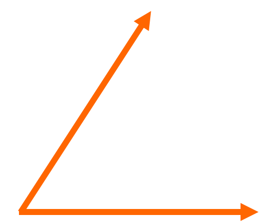


90 deg 60 deg 45 deg 30 deg

A **radian** is one $2\pi^{th}$ of a full revolution.

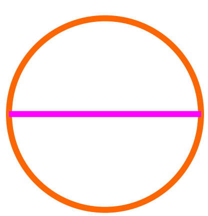
That is: 2π radians $=$ 360 degrees
π radians $=$ 180 degrees
1 radian $=$ $180/\pi$ degrees
$\pi/2$ radians $=$ 90 degrees
etc.

SKILL
rad/deg/rev conv



1 rad
($\approx$ 57 deg)

$\pi :=$ the ratio of the circumference of a circle to its diameter



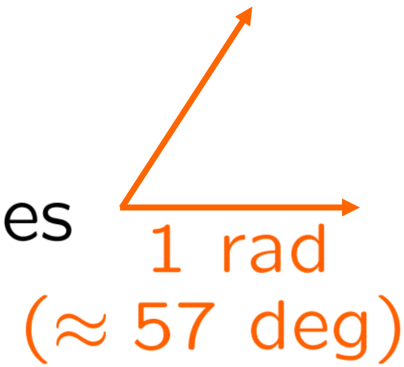
What are *DEGREE*s and *RADIAN*s?

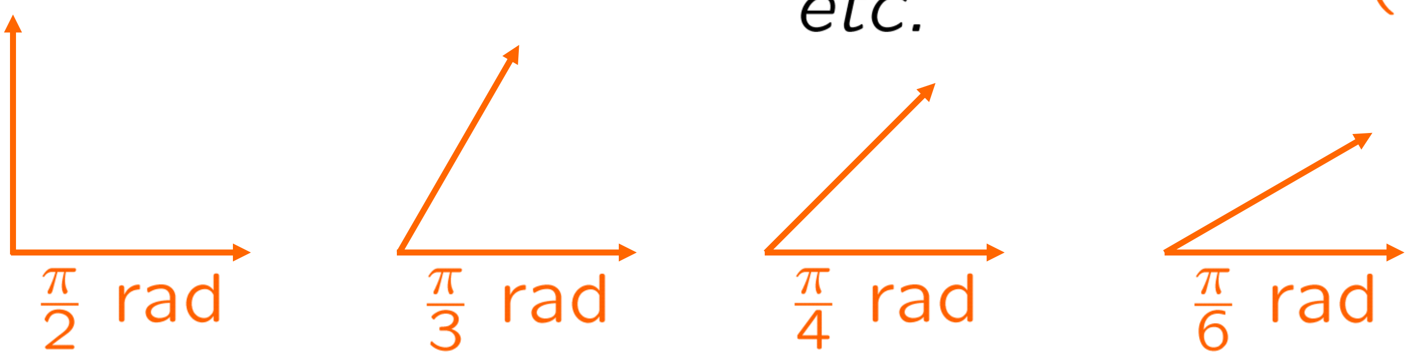
A **degree** is one 360^{th} of a full revolution.

A **radian** is one $2\pi^{th}$ of a full revolution.

$$\begin{aligned} \text{That is: } 2\pi \text{ radians} &= 360 \text{ degrees} \\ \pi \text{ radians} &= 180 \text{ degrees} \\ 1 \text{ radian} &= 180/\pi \text{ degrees} \\ \pi/2 \text{ radians} &= 90 \text{ degrees} \end{aligned}$$

etc.





In this class, we use radians, unless otherwise specified.

Why???

What are *DEGREE*s and *RADIAN*s?

A **degree** is one 360^{th} of a full revolution.
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$$1 \text{ radian} = 180/\pi \text{ degrees}$$

The **unit circle** is the circle of radius 1 whose center is the origin.

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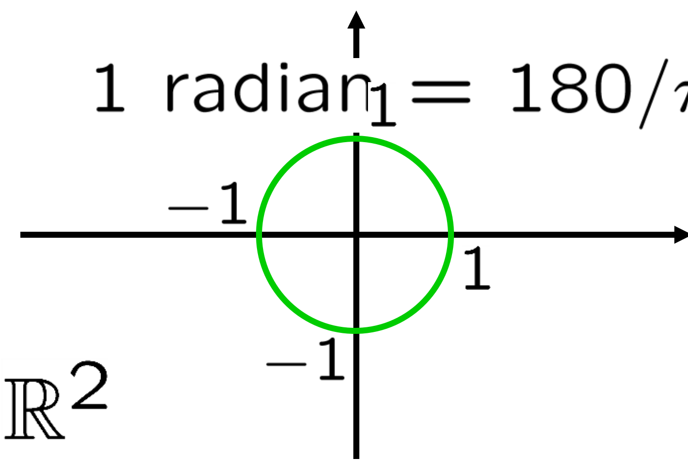

$\mathbb{R}^2$

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Why???

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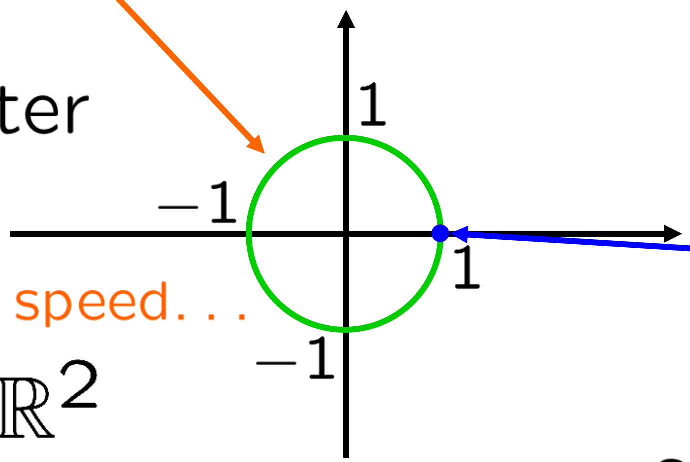
The **unit circle** is the circle of radius 1 whose center is the origin.

NOTE: Inaccurate, because 1 rev should take more than six seconds.

The **standard orbiter** is a particle that travels counterclockwise at speed 1 starting at $(1, 0)$

Circumference
$= \pi \times \text{diameter}$
$= 2\pi$

Magnify:
imagine uniform speed. . .



$\mathbb{R}^2$

1 rev per 2π secs
2π rads per 2π secs
1 rad per sec

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Why???

§4.1

What are *DEGREE*s and *RADIAN*s?

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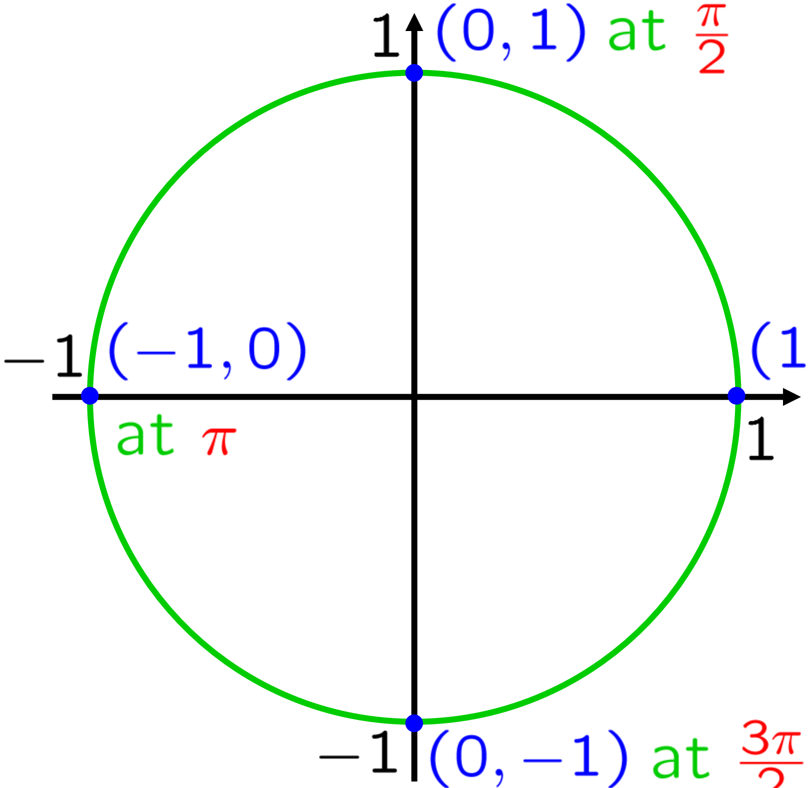

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1 rad per sec

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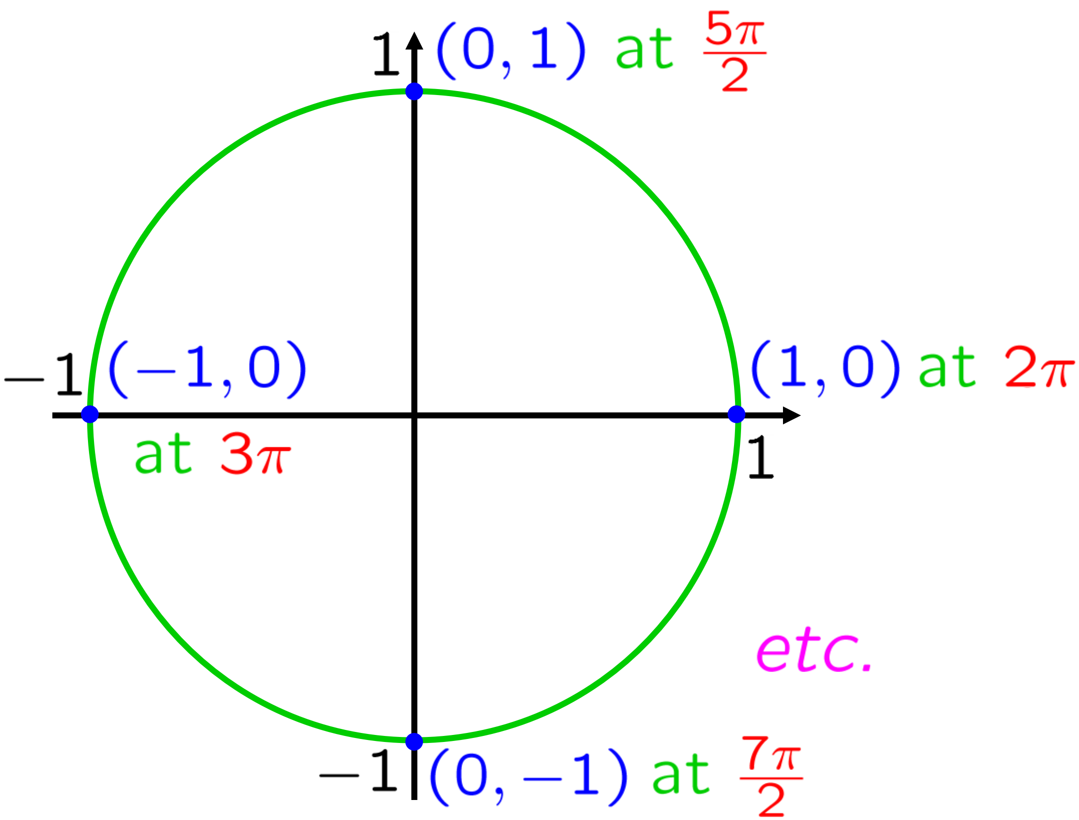

The **standard orbiter**
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1 rad per sec

Back to time 0. . .

What are *DEGREE*s and *RADIAN*s?

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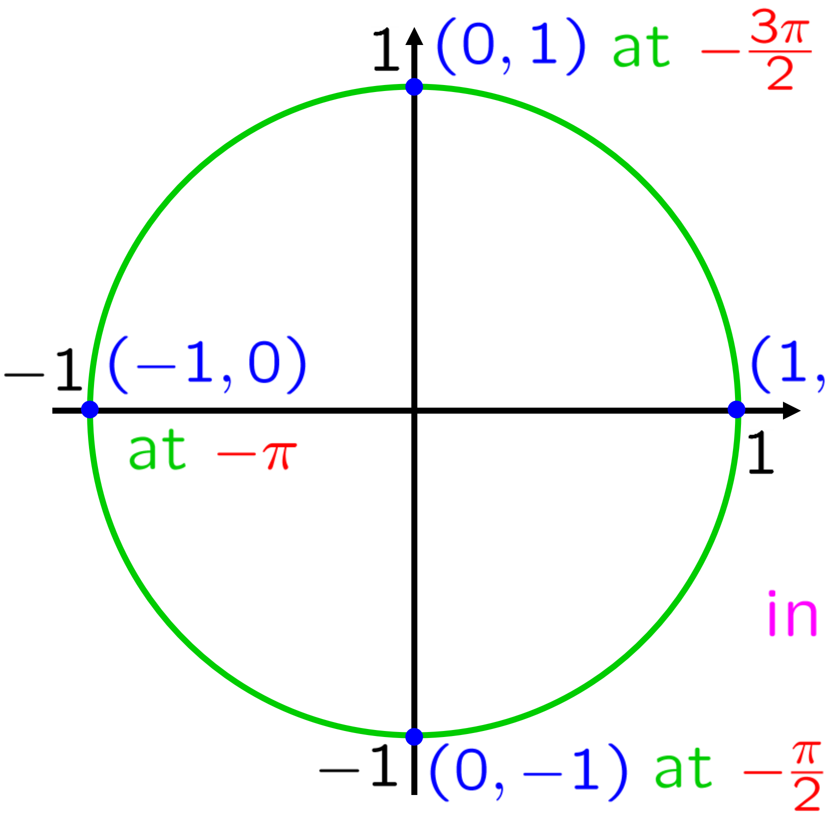

The **standard orbiter** is a particle that travels counterclockwise at speed 1 starting at $(1, 0)$

in reverse time . . .

1 rad per sec

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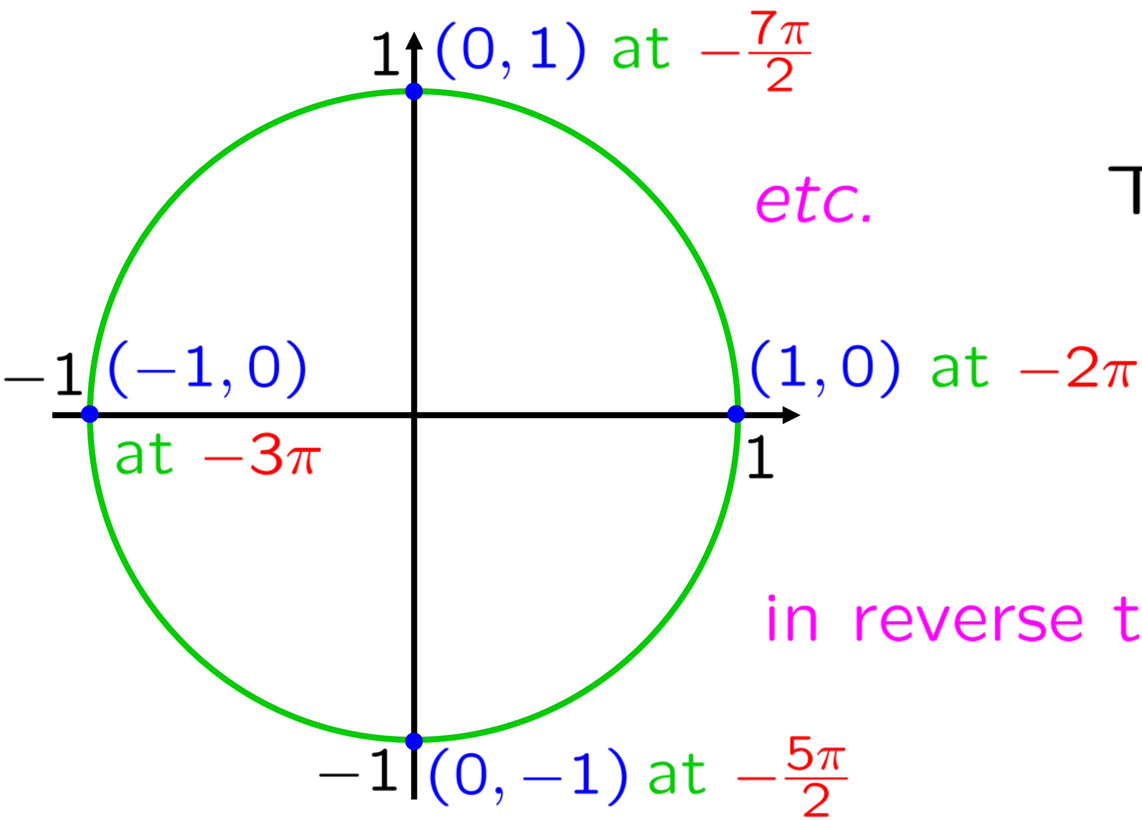

The **standard orbiter** is a particle that travels counterclockwise at speed 1 starting at (1, 0)

1 rad per sec

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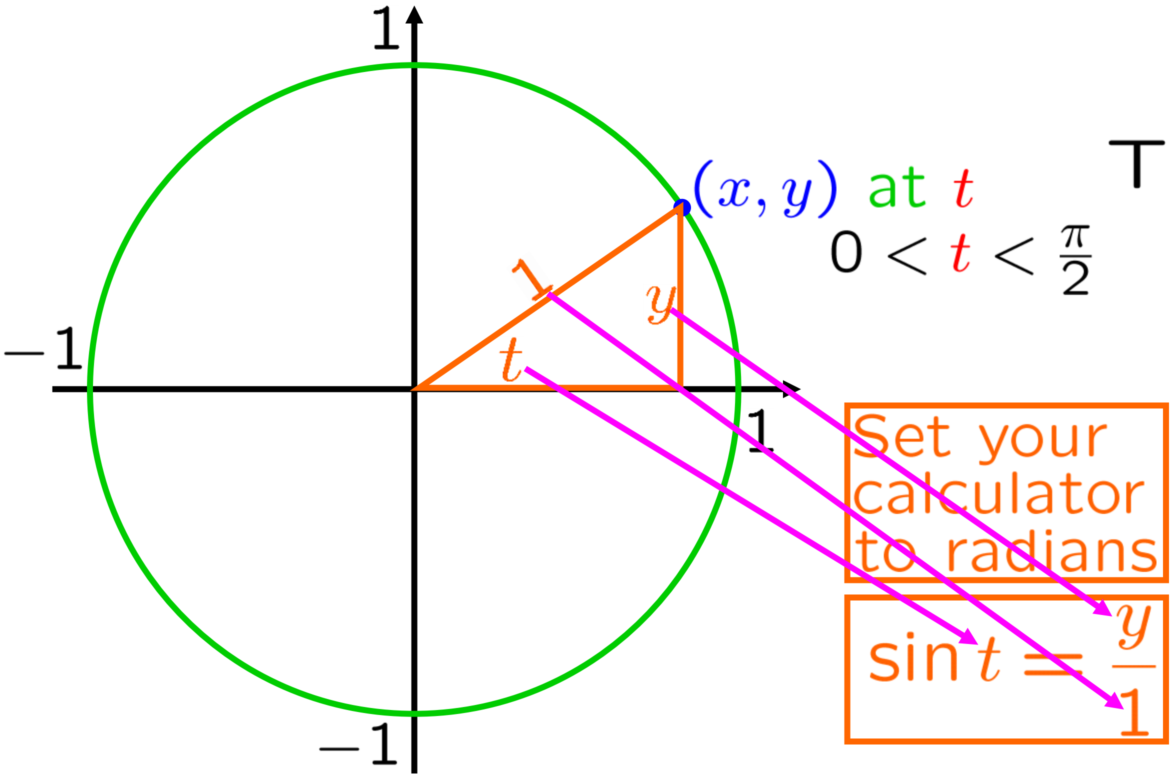

The **standard orbiter** is a particle that travels counterclockwise at speed 1 starting at $(1, 0)$

. . . but that's like oh, so high school!

1 rad per sec

What is sine in this class?

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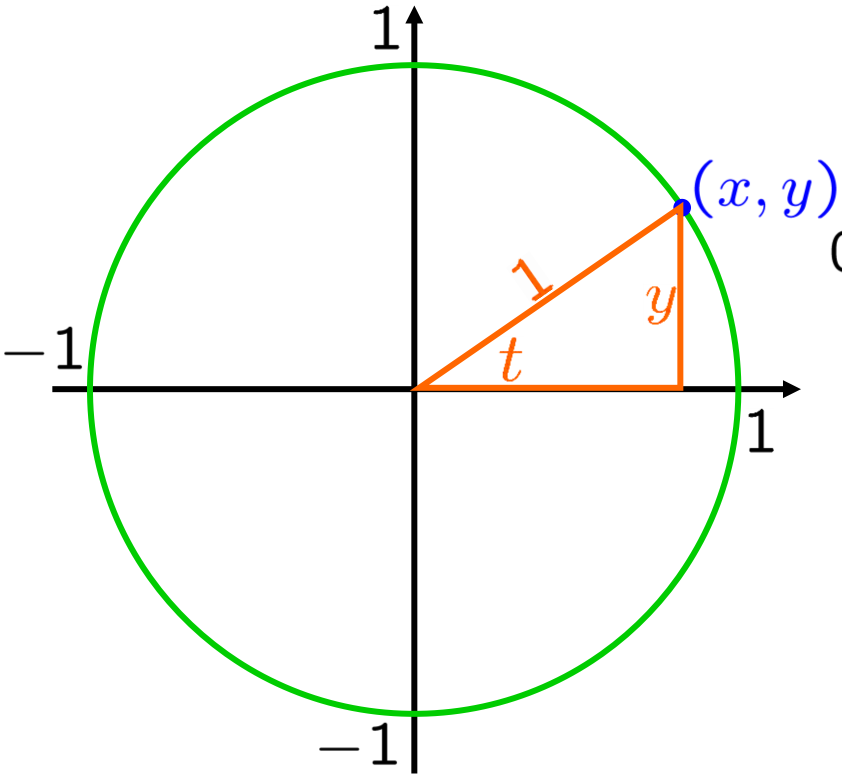


$0 < t < \frac{\pi}{2}$

The **standard orbiter** is a particle that travels counterclockwise at speed 1 starting at $(1, 0)$

Set your calculator to radians

$\sin t = \frac{y}{1}$

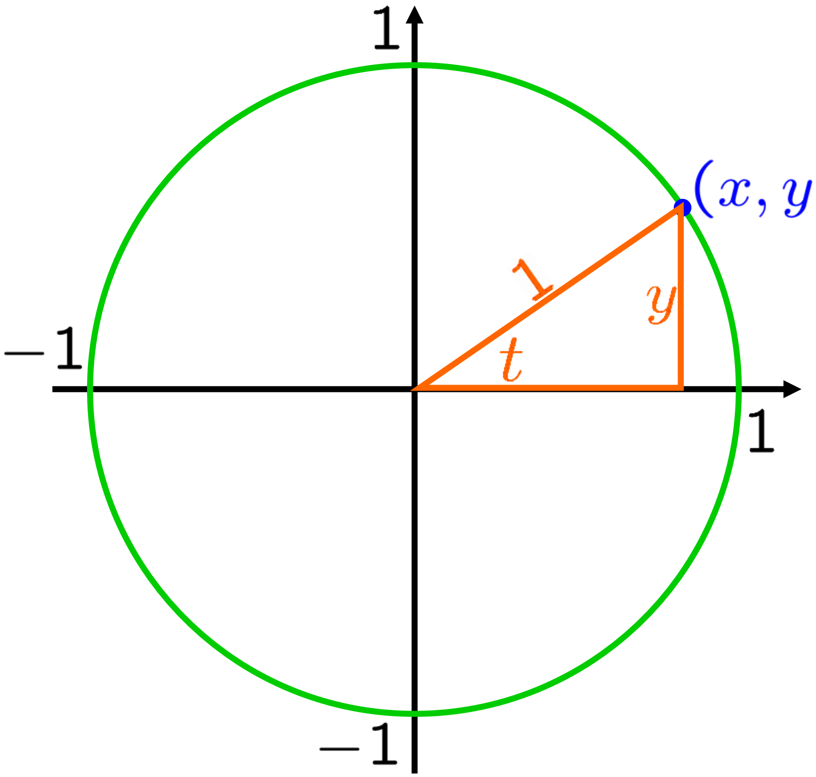
. . . but that's like oh, so high school!

1 rad per sec

What is sine in this class?

t can be any (real) number . . . very college!

$\forall t \in \mathbb{R}$, let $\sin t$ be the y-coordinate of the standard orbiter at time t



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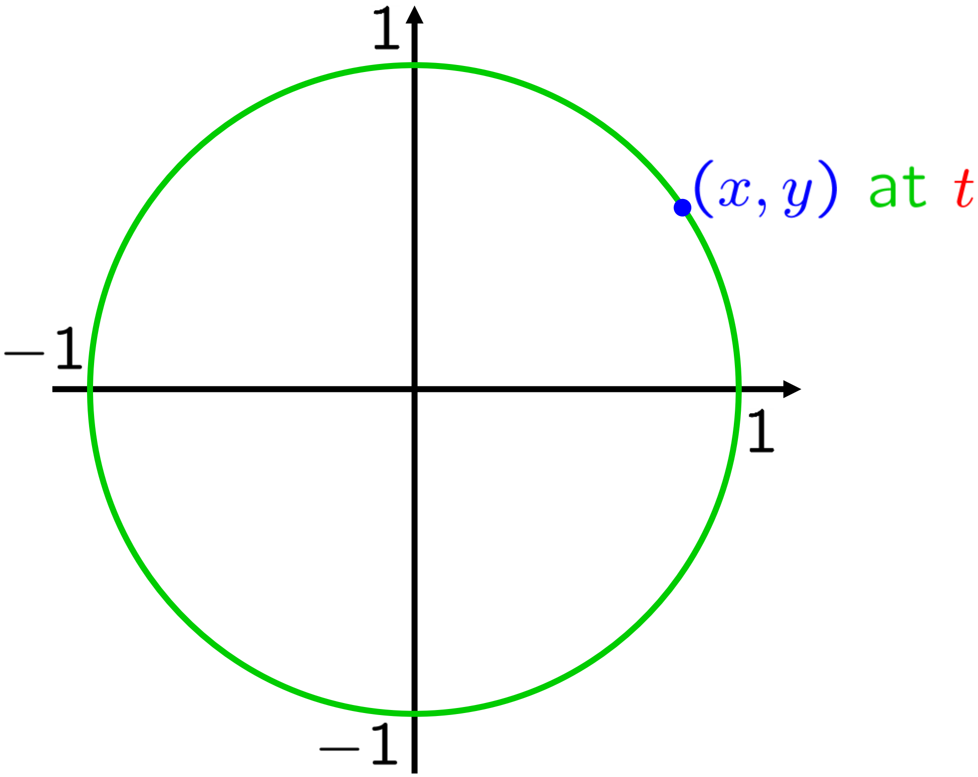
What is sine in this class?

$\forall t \in \mathbb{R}$, let $\boxed{\sin t}$ be the y-coordinate of the standard orbiter at time t

sin is periodic with period 2π



t	$\sin t$
$\frac{\pi}{2}$	$\frac{\sqrt{4}}{2} = 1$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{5}$	SKIP
$\frac{\pi}{6}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
0	$\frac{\sqrt{0}}{2} = 0$

Proofs...

§4.1

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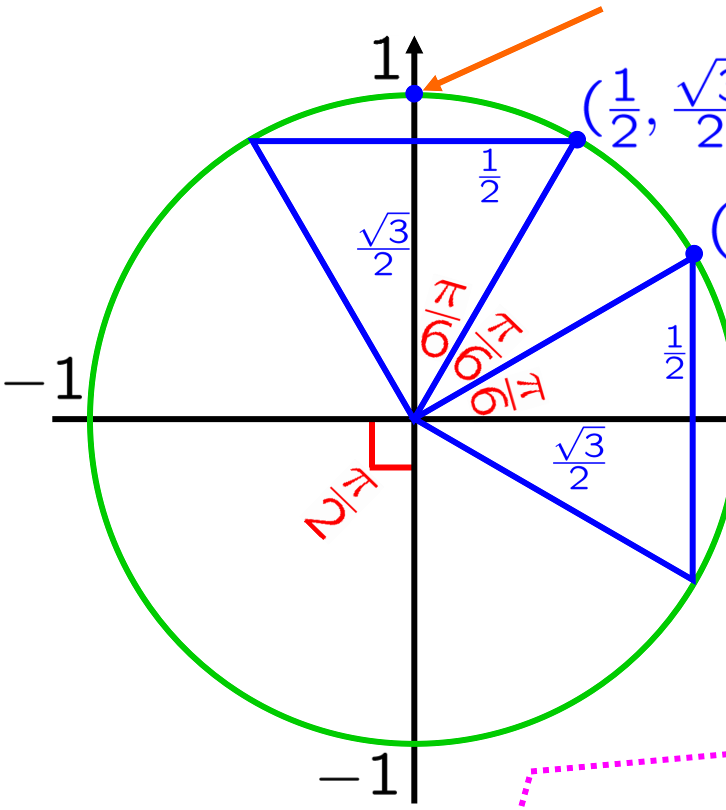
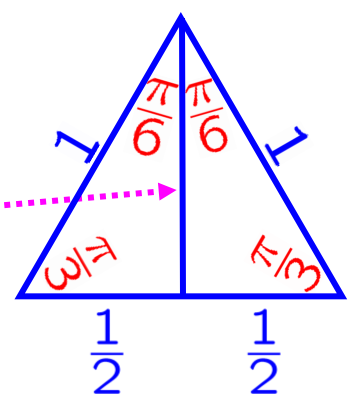


$$\frac{\sqrt{3}}{2} = \sqrt{1^2 - [\tfrac{1}{2}]^2}$$



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Proofs...

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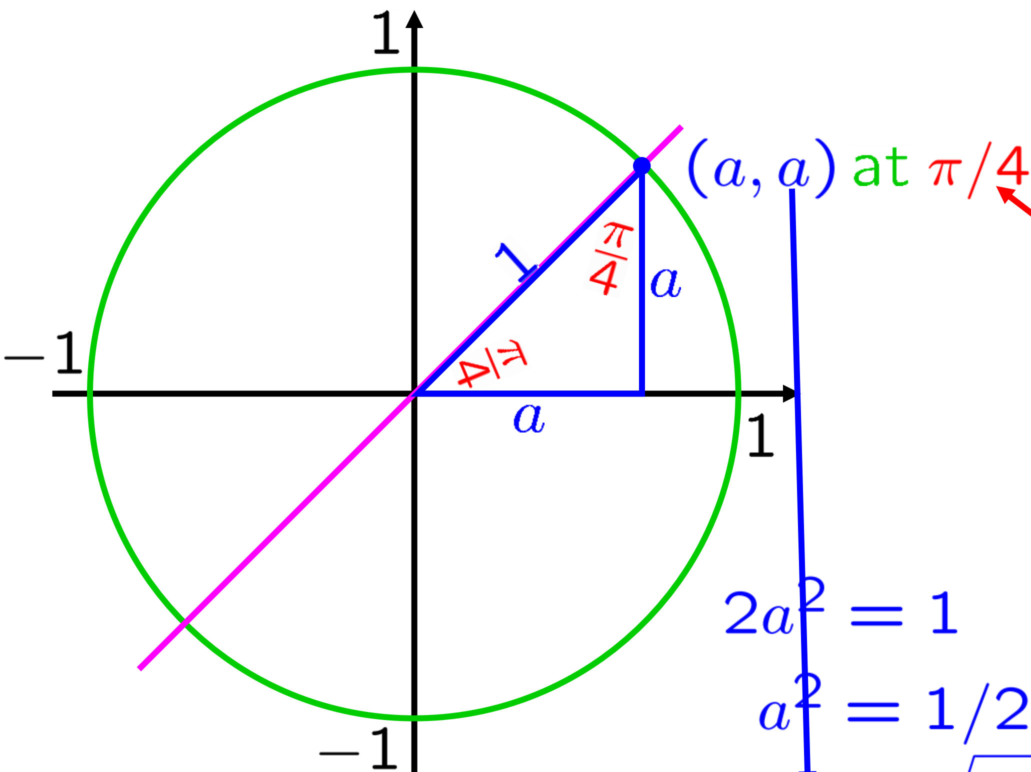

Goal: Find a.

$$a^2 + a^2 = 1^2$$

$$2a^2 = 1$$

$$a^2 = 1/2$$

$$a = \sqrt{1/2}$$

$$= \sqrt{2}/2$$

t	$\sin t$
$\frac{\pi}{2}$	$\frac{\sqrt{4}}{2} = 1$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
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Proofs...

SKILL
sin comp

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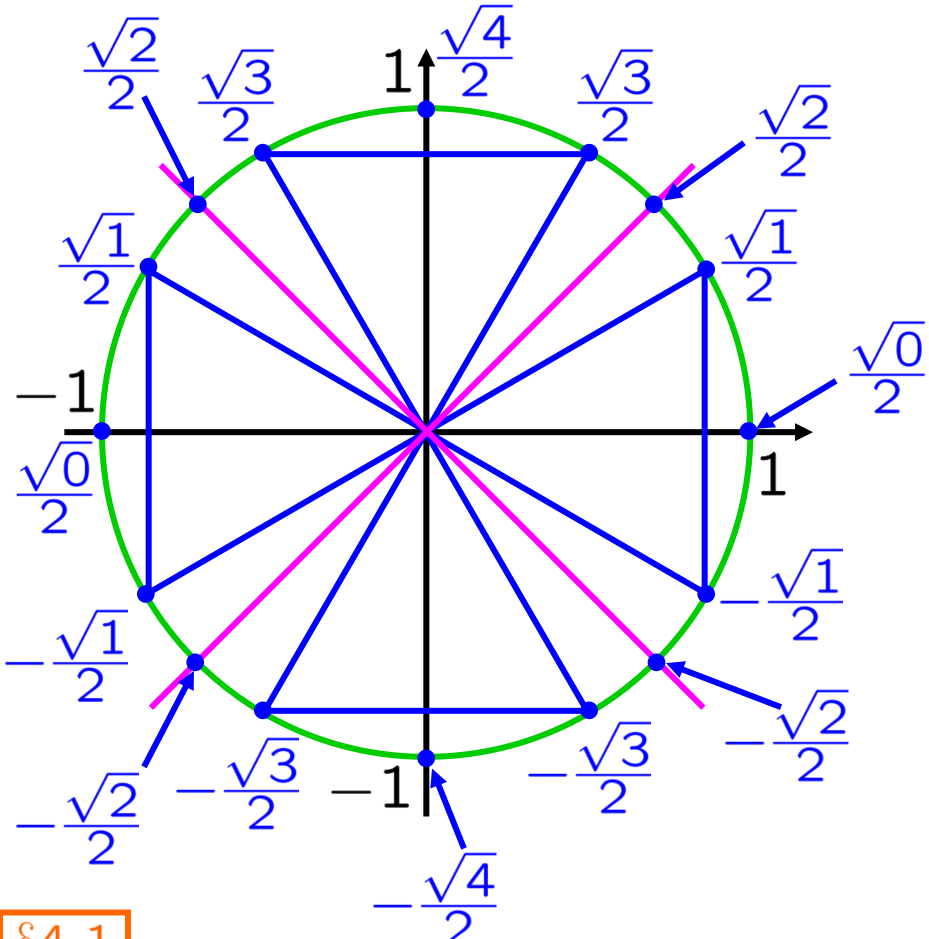

SKILL
sin comp

Learn the y-coordinate of all of these blue points on the circle

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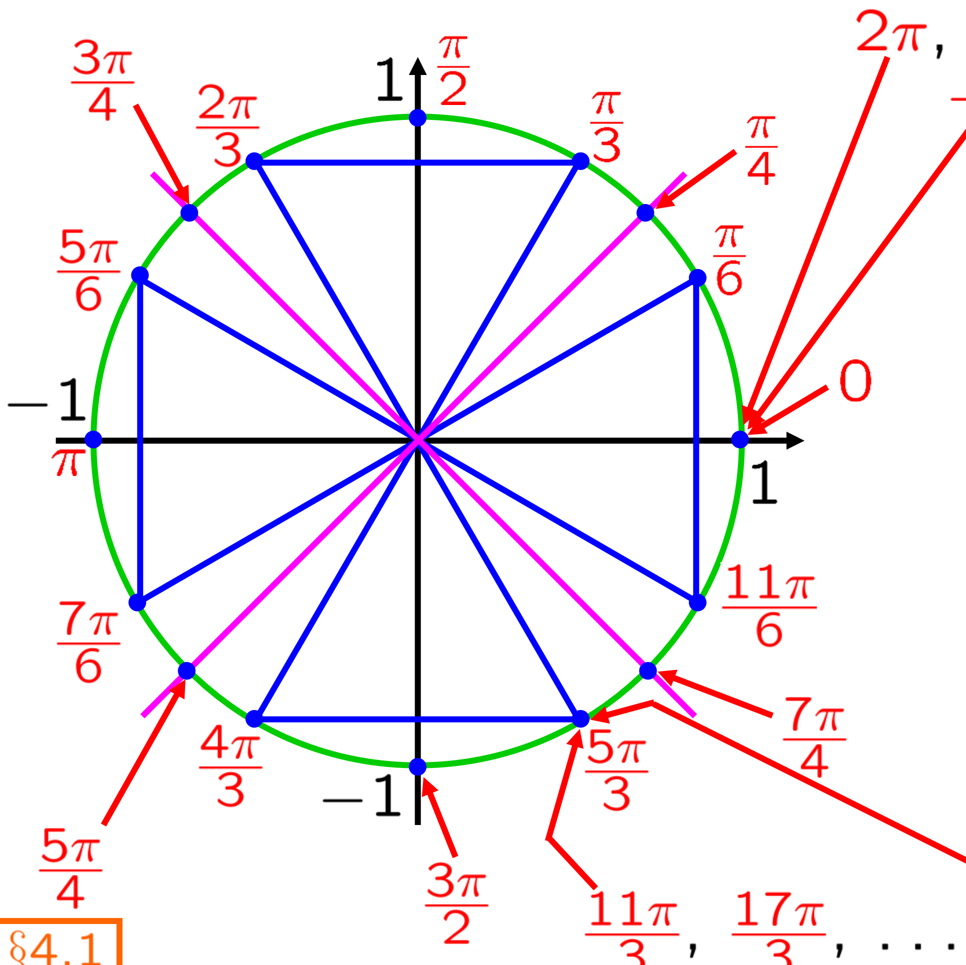


SKILL
sin comp

Learn the times of all of these blue points on the circle . . .

Remember to SKIP $\frac{\pi}{5}$.

ADD $\frac{\pi}{2}$.

ADD another $\frac{\pi}{2}$.

ADD another $\frac{\pi}{2}$.

REMEMBER 2π-periodicity!

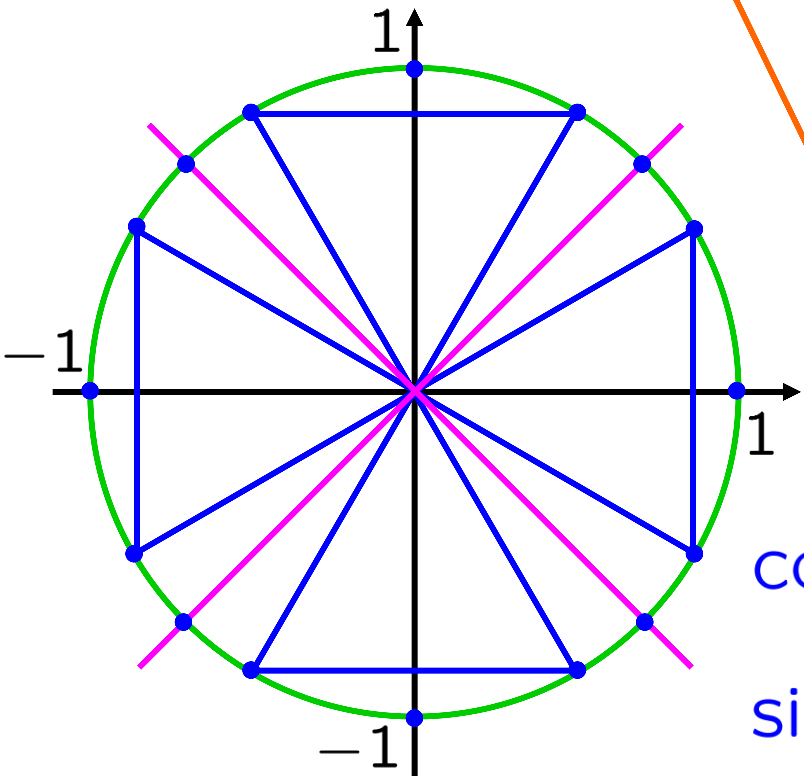
etc.

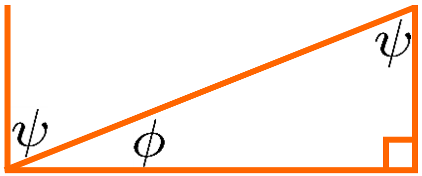
What is cosine in this class?

$\forall t \in \mathbb{R}$, let $\boxed{\sin t}$ be the y-coordinate of the standard orbiter at time t

$\forall t \in \mathbb{R}$, let $\boxed{\cos t} := \sin(\frac{\pi}{2} - t)$.

Two angles are **complementary** if their sum is $\frac{\pi}{2}$.





$\phi + \psi = \frac{\pi}{2}$

cos of an angle is defined to be sin of the complementary angle.

What is cosine in this class?

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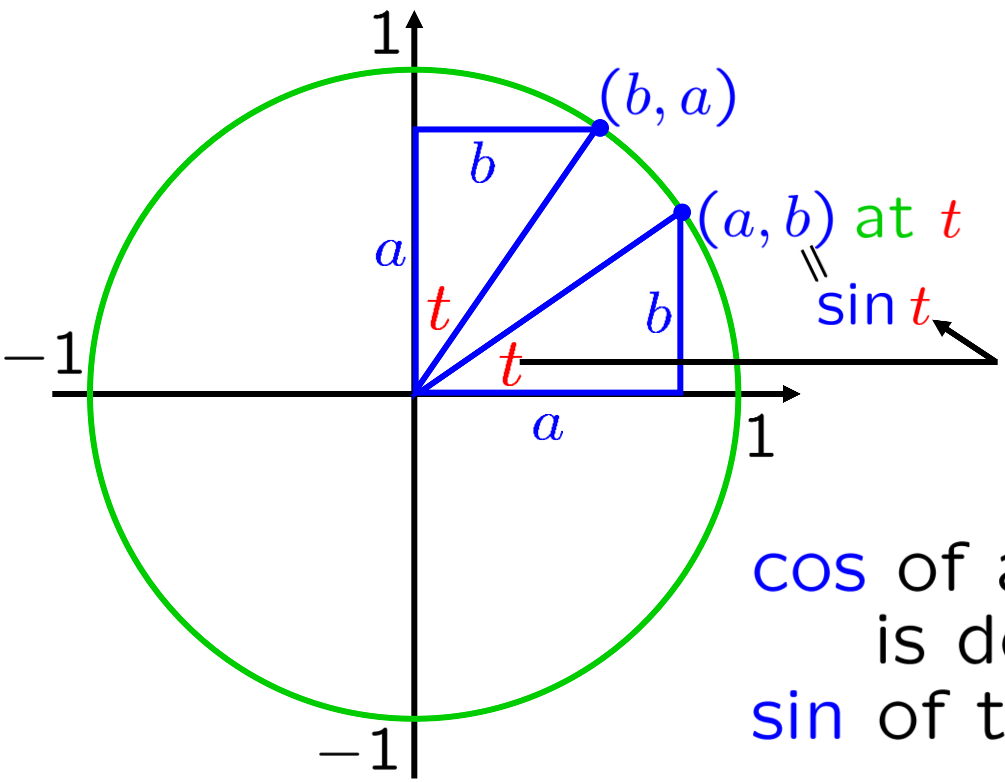

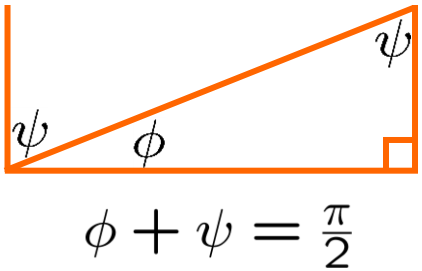

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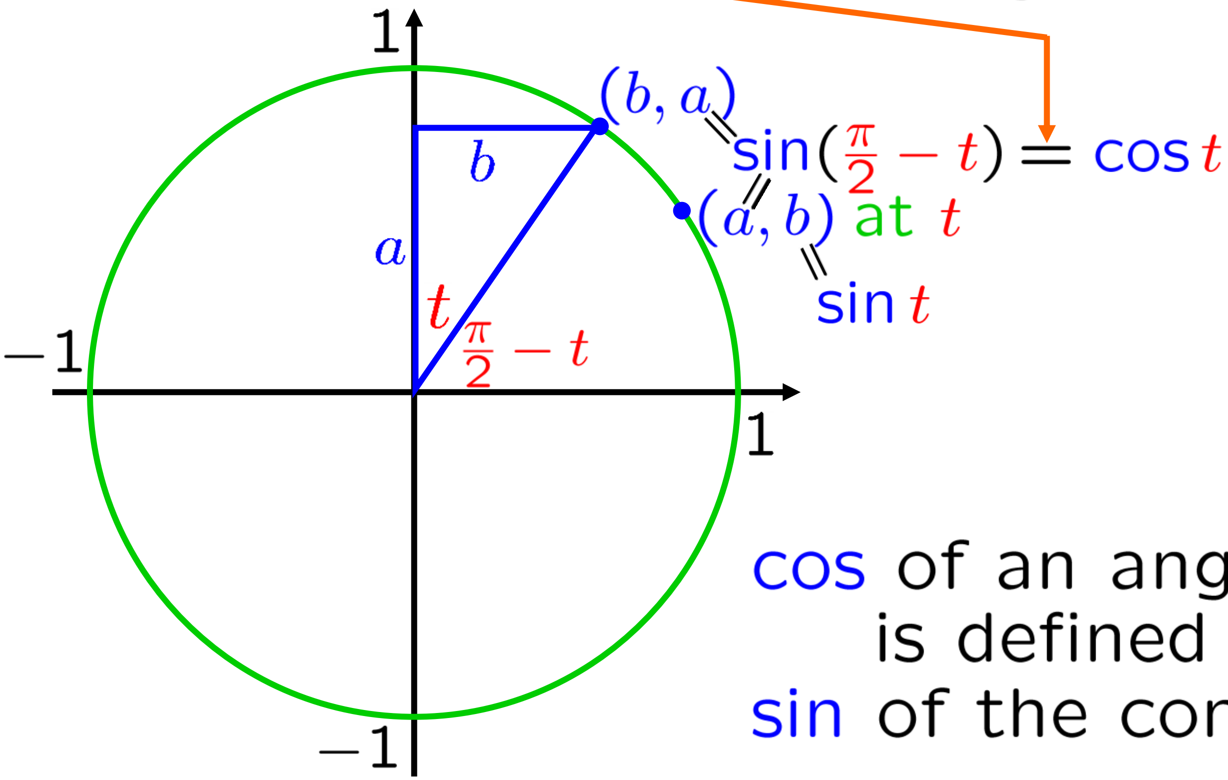

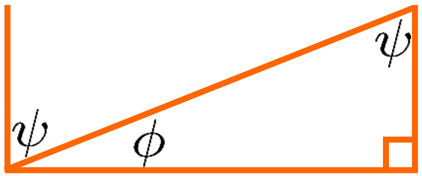

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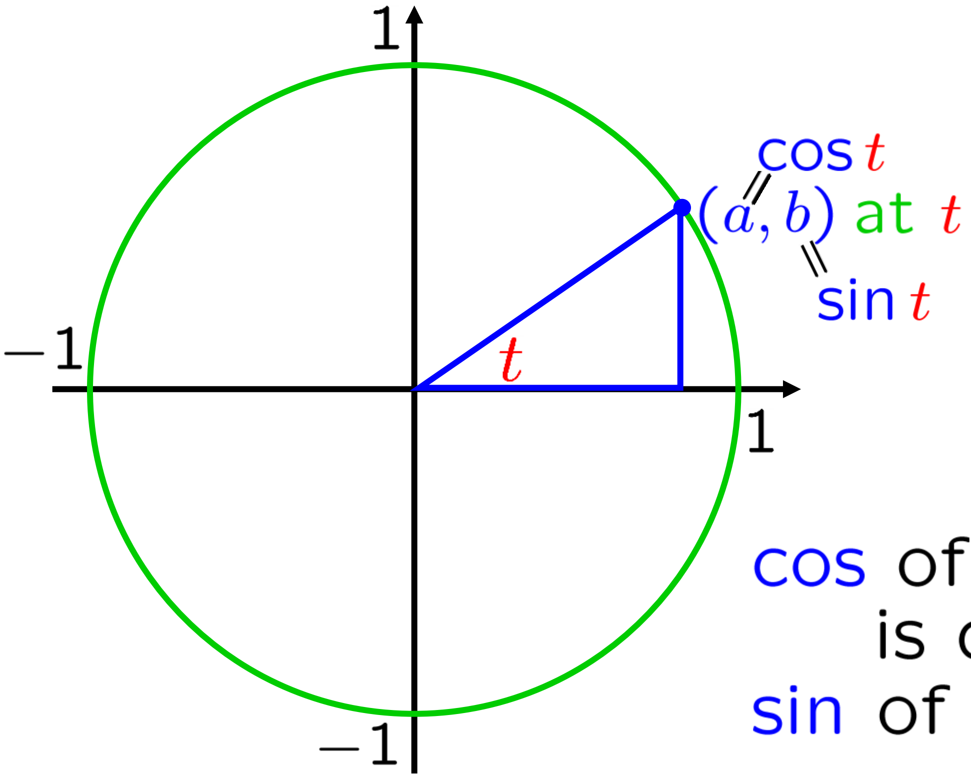
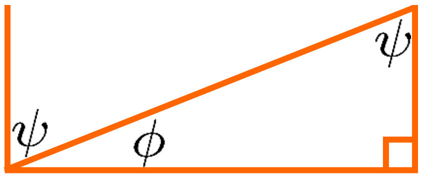


cos t



$\phi + \psi = \frac{\pi}{2}$

cos of an angle
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$\forall t \in \mathbb{R}$, $\cos t$ is the x-coordinate of
the standard orbiter at time t

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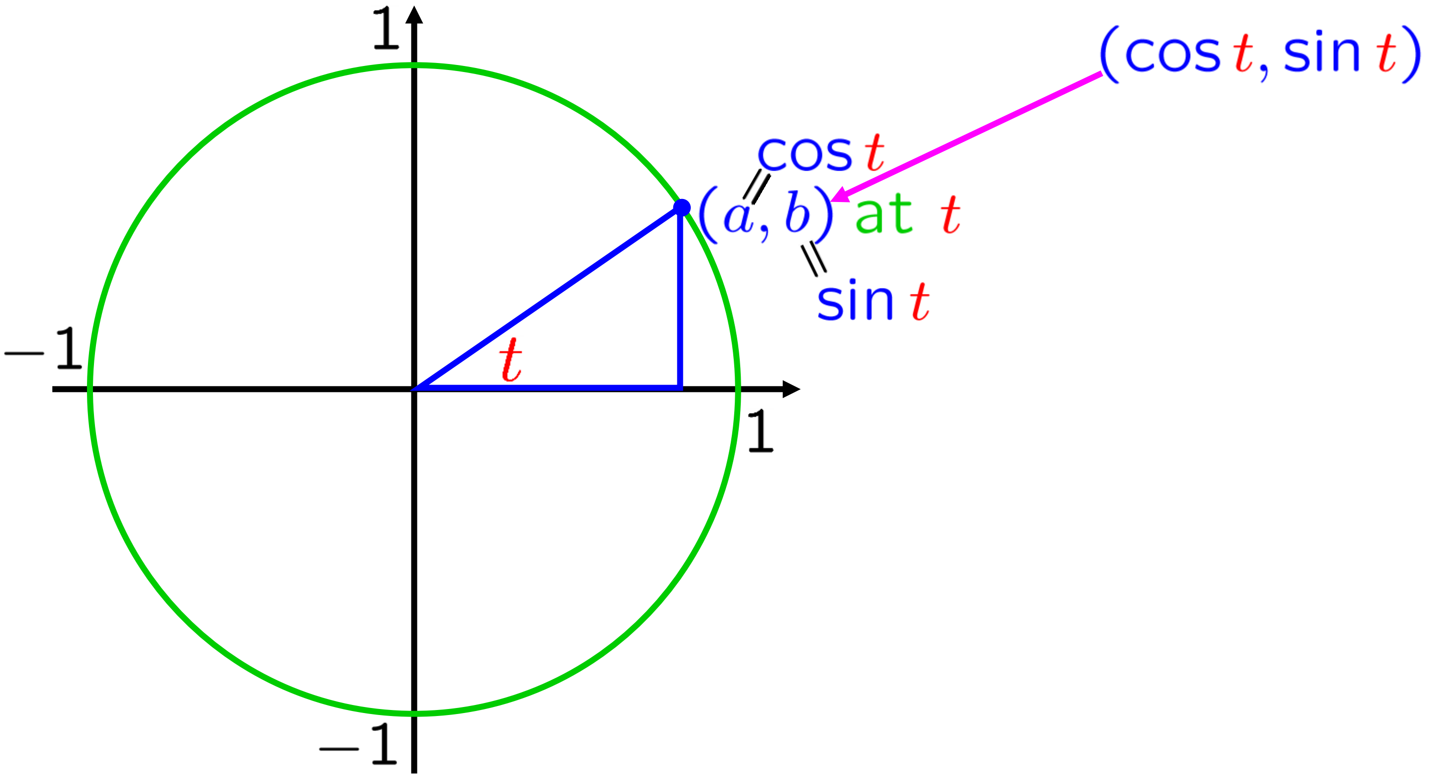

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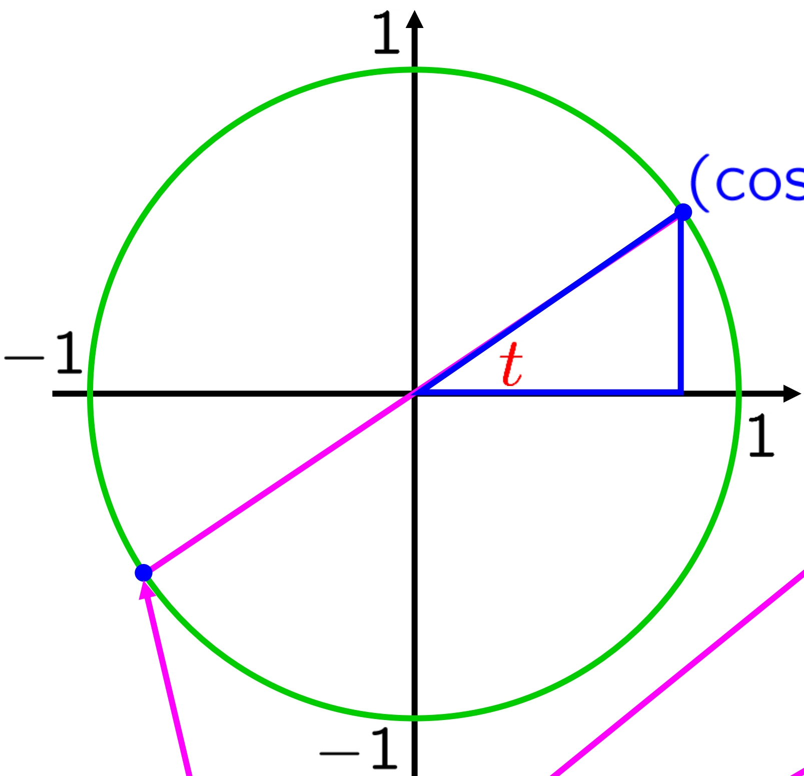
Next: π-antiperiodicity of sin and cos



$(\cos t, \sin t)$ at t

$\forall t \in \mathbb{R}$,

$$\cos(t+\pi) = -\cos t$$
$$\sin(t+\pi) = -\sin t$$

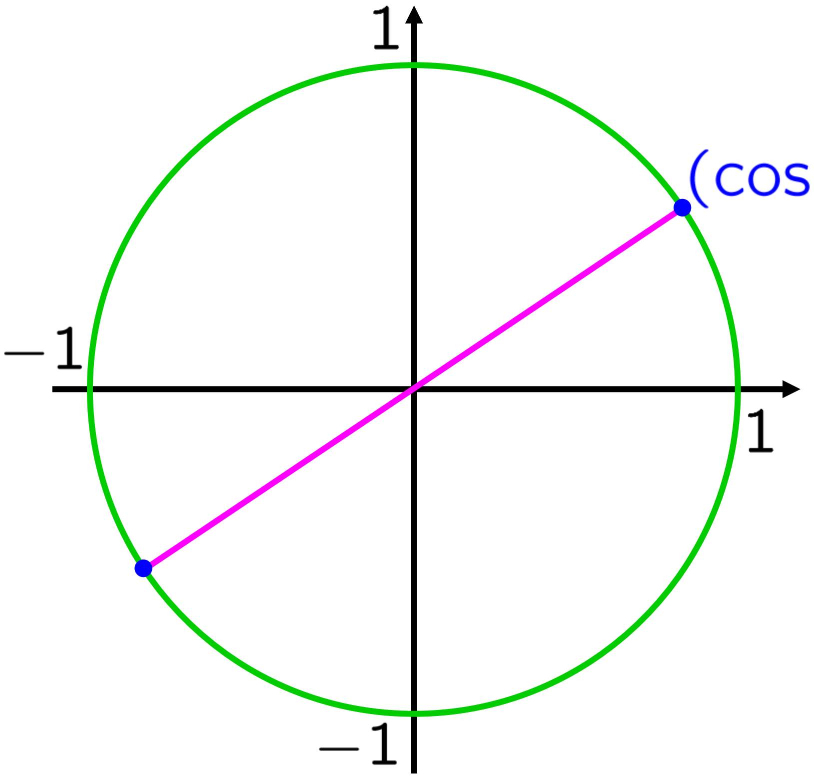
t is a "dummy variable"

$(\cos(t+\pi), \sin(t+\pi))$ at $t+\pi$

$= (-\cos t, -\sin t)$

What is cosine in this class?

$\forall t \in \mathbb{R}$, $\cos t$ is the x-coordinate of the standard orbiter at time t



$\forall t \in \mathbb{R}$,

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t is a "dummy variable"

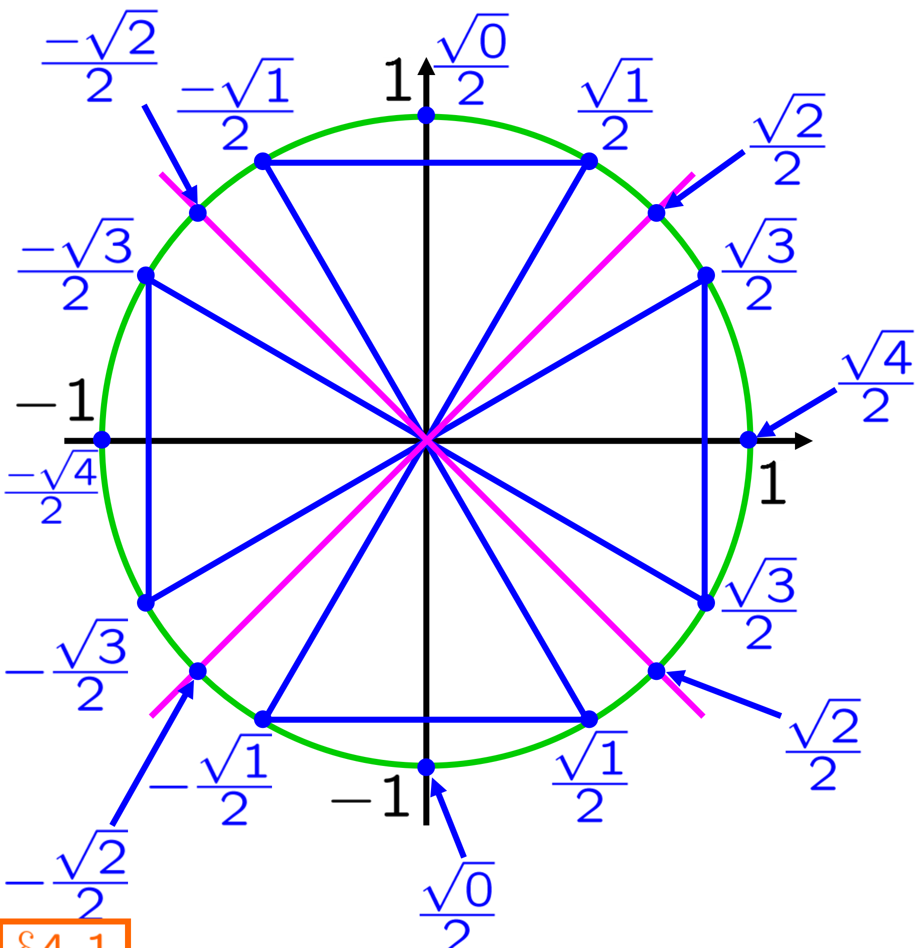
$\forall x \in \mathbb{R}$,

$$\cos(x+\pi) = -\cos x$$
$$\sin(x+\pi) = -\sin x$$

Learn these formulas!

What is cosine in this class?

$\forall t \in \mathbb{R}$, $\cos t$ is the x-coordinate of the standard orbiter at time t

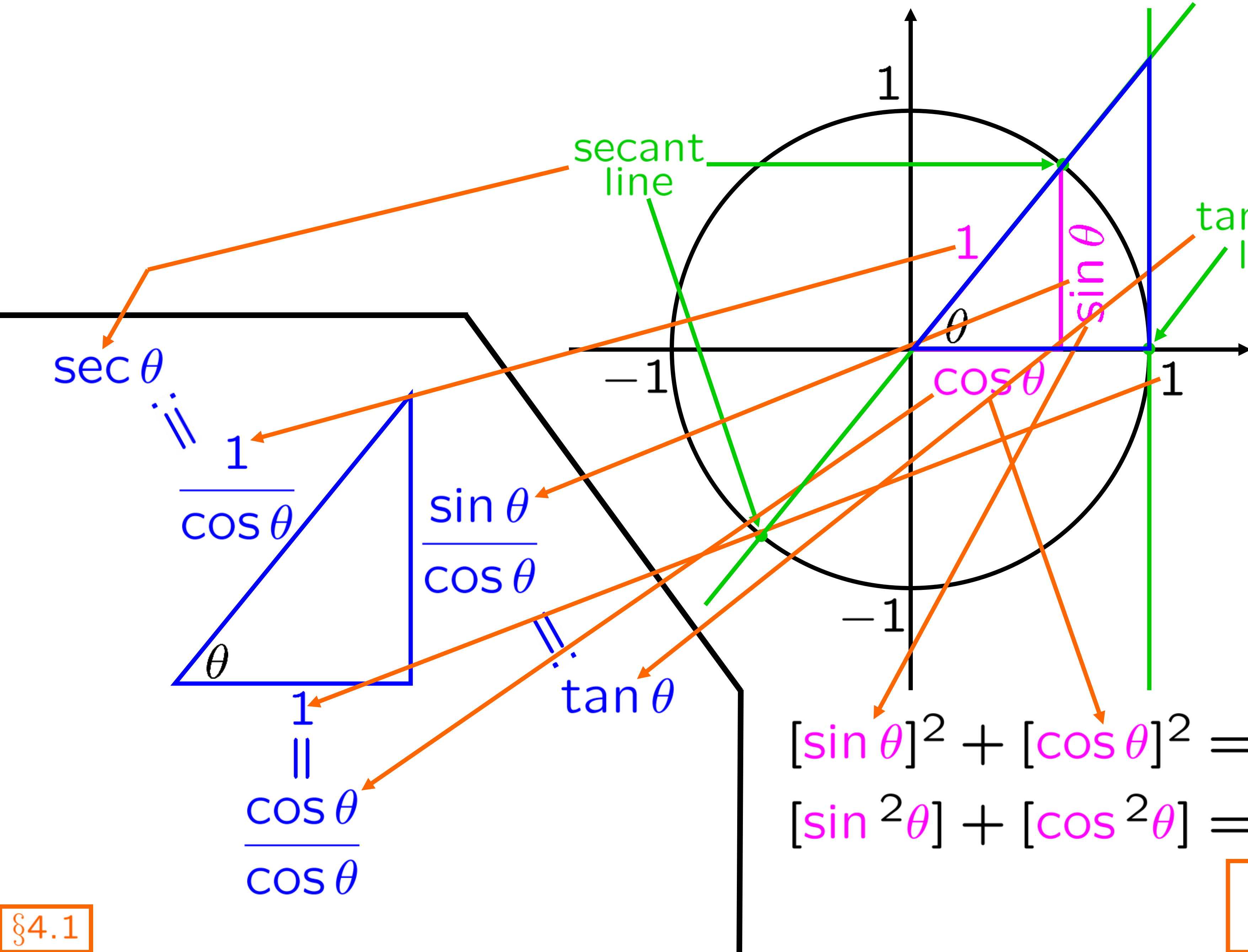


SKILL
cos comp

Learn the x-coordinate of all of these blue points on the circle . . .

§4.1

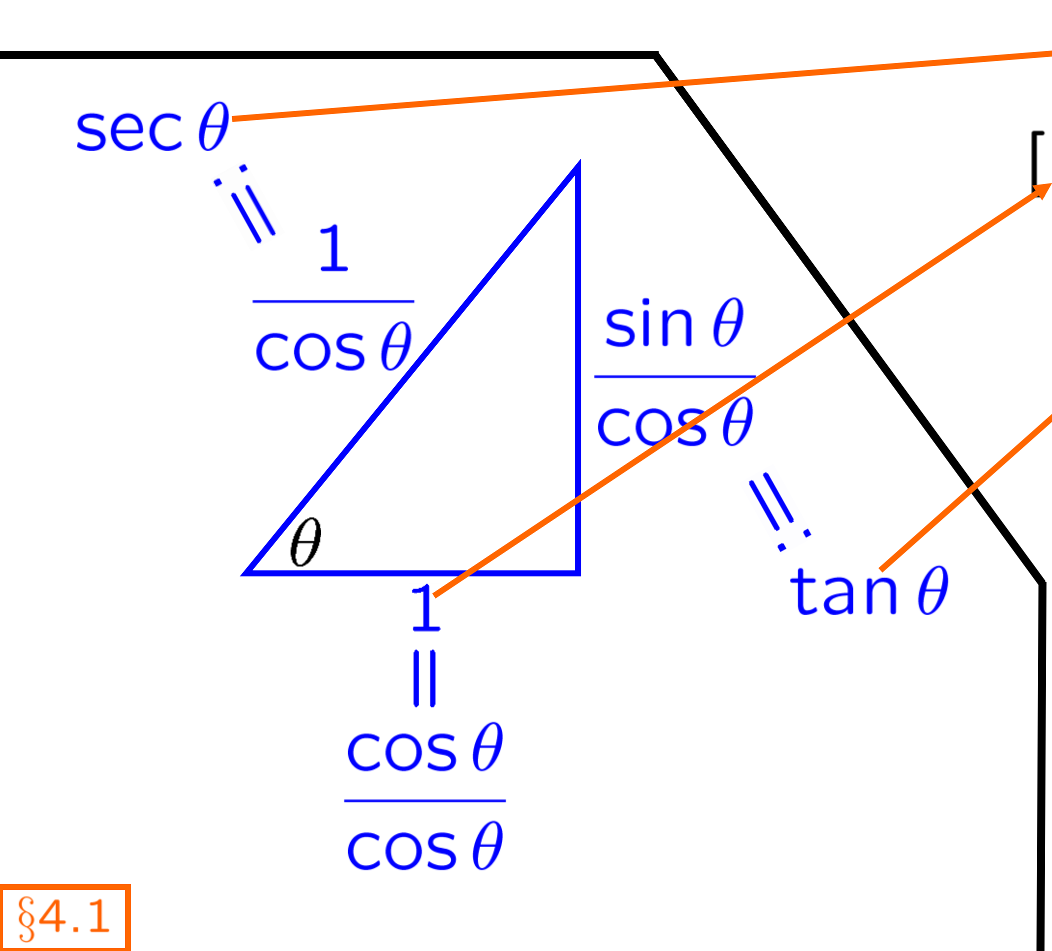
New functions & formulas...



$$[\sin\theta]^2 + [\cos\theta]^2 = [1]^2$$

$$[\sin^2\theta] + [\cos^2\theta] = 1$$

New functions & formulas. . .



$$[1]^2 + [\tan\theta]^2 = [\sec\theta]^2$$

$$1 + [\tan^2\theta] = \sec^2\theta$$

$$[\sin\theta]^2 + [\cos\theta]^2 = [1]^2$$

$$[\sin^2\theta] + [\cos^2\theta] = 1$$

New functions & formulas...

tan, sec

complementary angles

$$\cos\theta := \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta := \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta := \frac{1}{\cos\theta}$$

$$\sec\theta \stackrel{\cdot\cdot}{=} \frac{1}{\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta} \stackrel{\cdot\cdot}{=} \tan\theta$$

$$1 + [\tan^2\theta] = \sec^2\theta$$

$$1 + [\tan^2\theta] = \sec^2\theta$$

$$[\sin^2\theta] + [\cos^2\theta] = 1$$

New functions & formulas...

tan, sec, cot, csc

complementary angles

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta := \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta := \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta := \sin(\tfrac{\pi}{2} - \theta)$$

$$\cot\theta := \tan(\tfrac{\pi}{2} - \theta)$$

$$\csc\theta := \sec(\tfrac{\pi}{2} - \theta)$$

Learn all these formulas!

$$\theta :\longrightarrow \tfrac{\pi}{2} - \theta$$

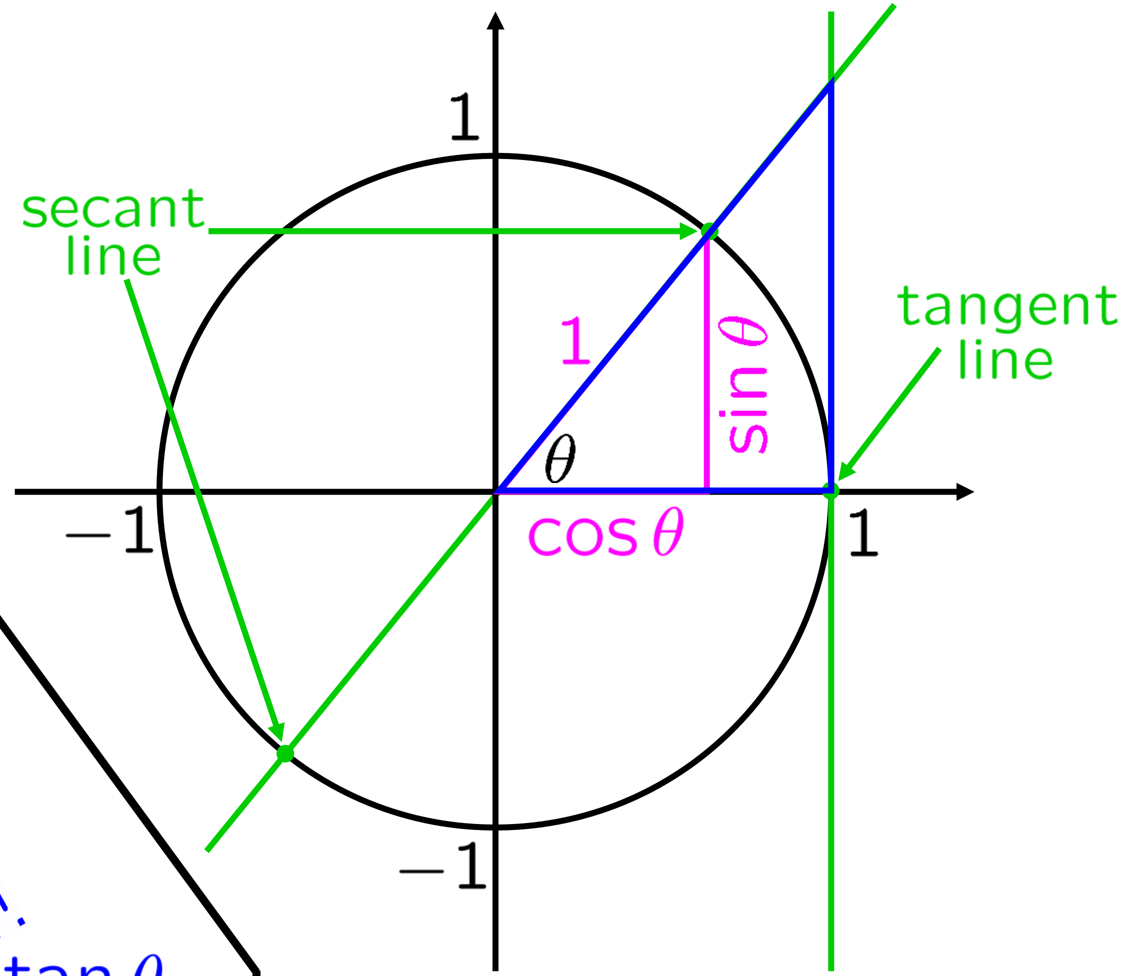
$$1 + [\cot^2\theta] = \csc^2\theta$$

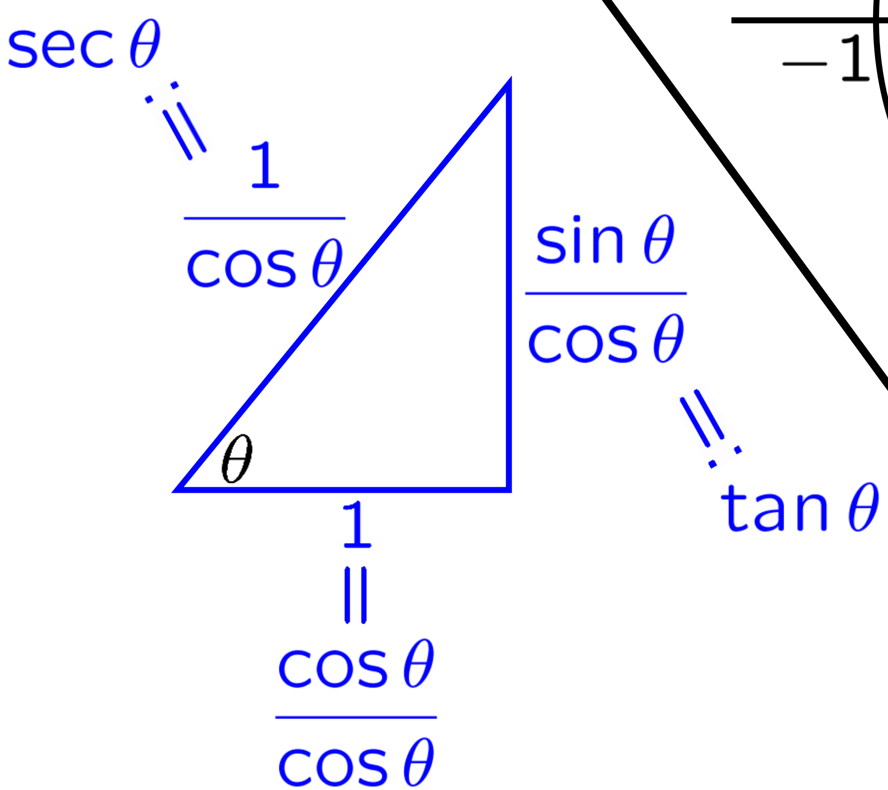
$$1 + [\tan^2\theta] = \sec^2\theta$$

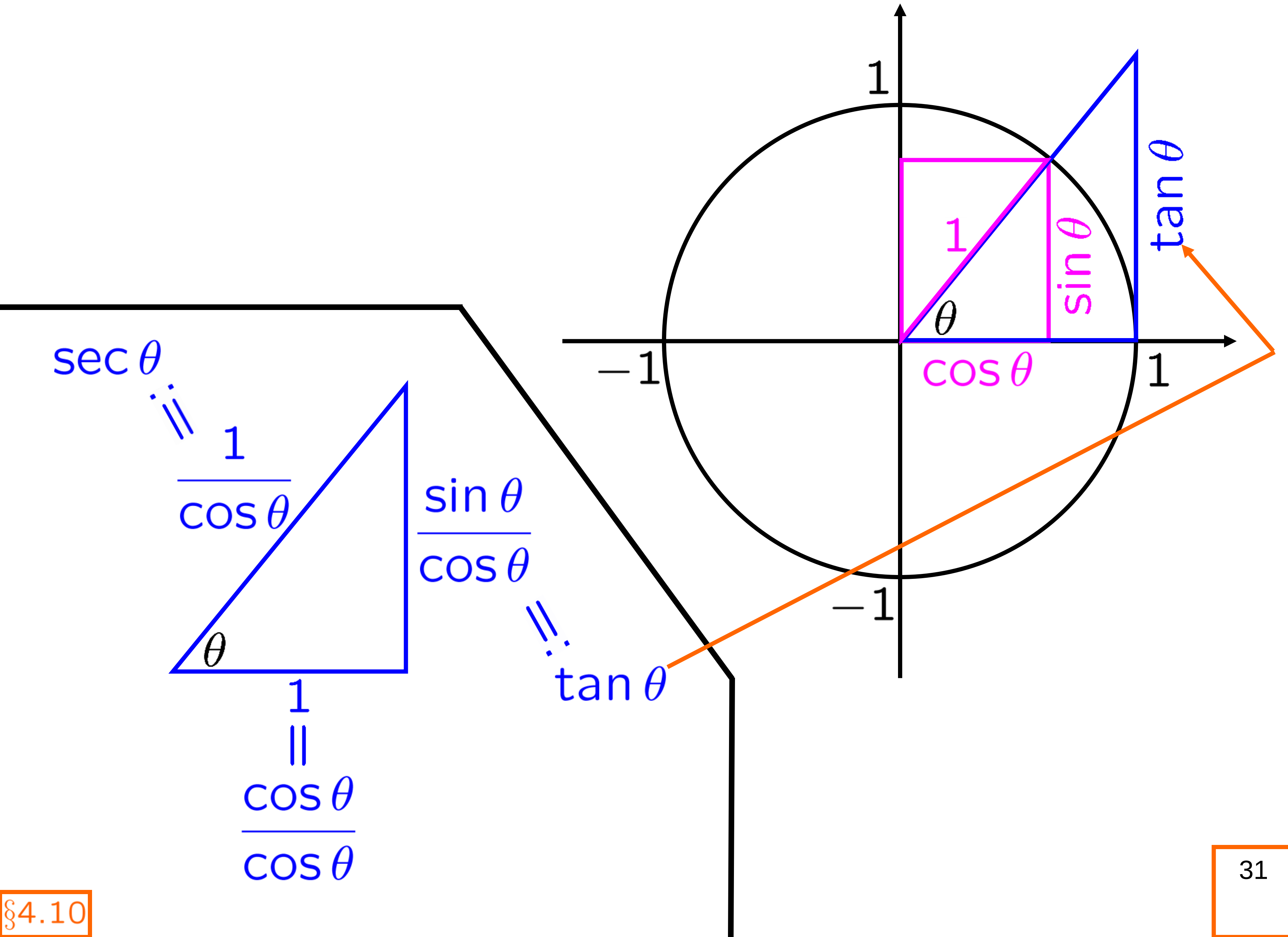
$$[\sin^2\theta] + [\cos^2\theta] = 1$$

self-complementary

Recall. . .


1
secant
line
tangent
line
1
sin θ
θ
−1
cos θ
1
−1


sec θ
=
1
cos θ
sin θ
cos θ
=
tan θ
θ
1
=
cos θ
cos θ


sec θ
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1
−1
1
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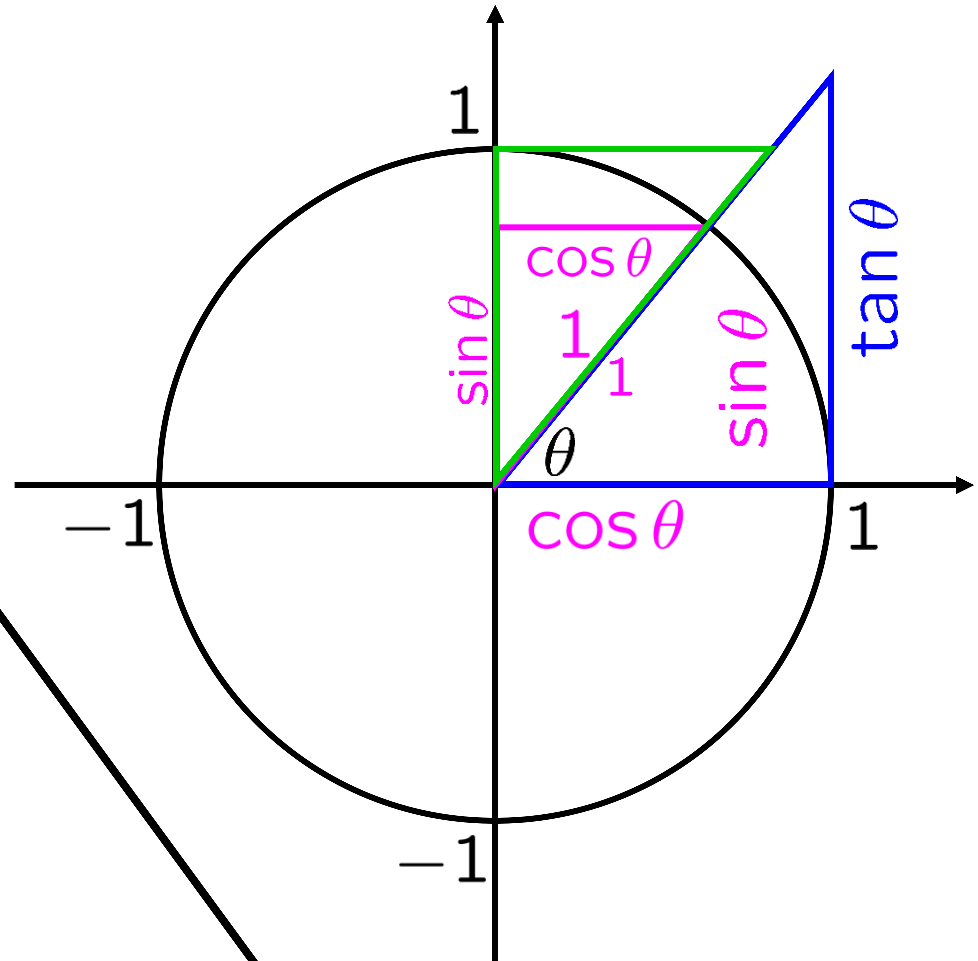
θ
sin θ
cos θ
tan θ


1
cos θ
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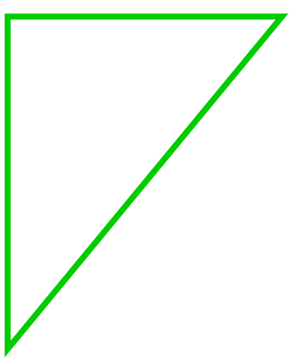
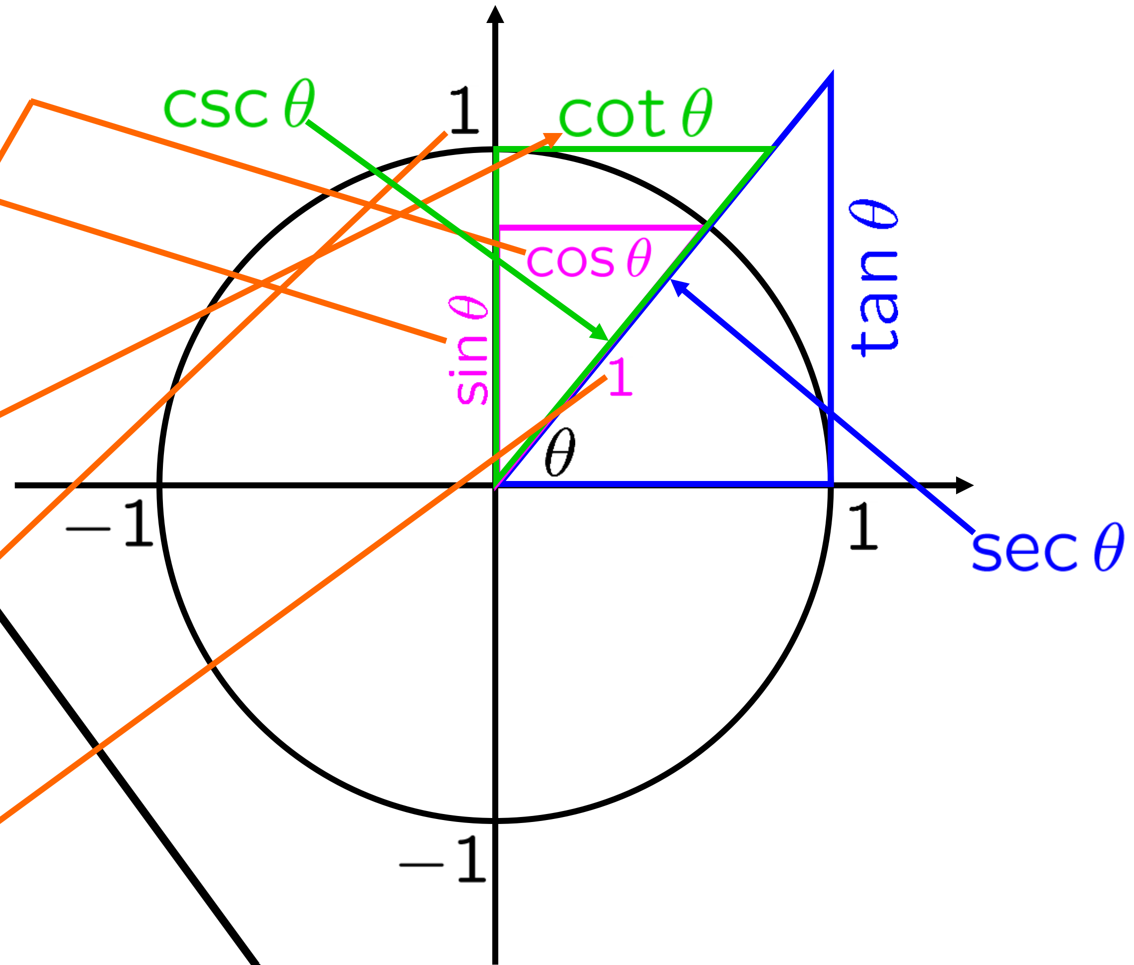
tan θ
θ
−1
cos θ
1
−1



The geometry of the six trig functions:



$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin\theta}{\sin\theta} = 1 \qquad \frac{1}{\sin\theta} = \csc\theta$$

SKILL
preimage under trig

Whitman problems
§4.1, p. 66, #1-2

SKILL
image under trig

Whitman problems
§4.1, p. 66, #3-4

SKILL
trig identities

Whitman problems
§4.1, p. 66, #5-7

SKILL
trig graphing

Whitman problems
§4.1, p. 66, #8-10

SKILL
solve trig eq'n

Whitman problems
§4.1, p. 66, #11

STOP