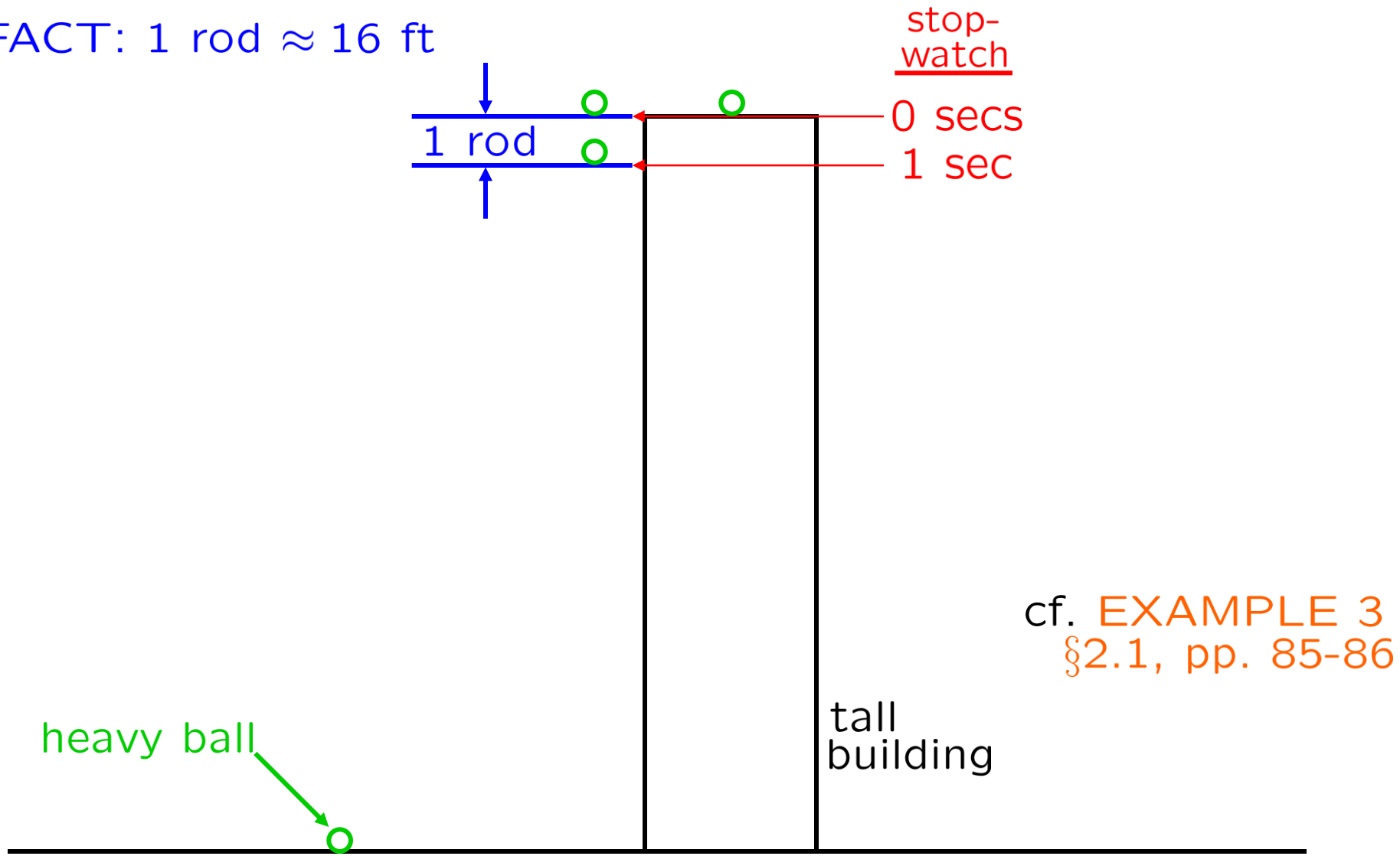
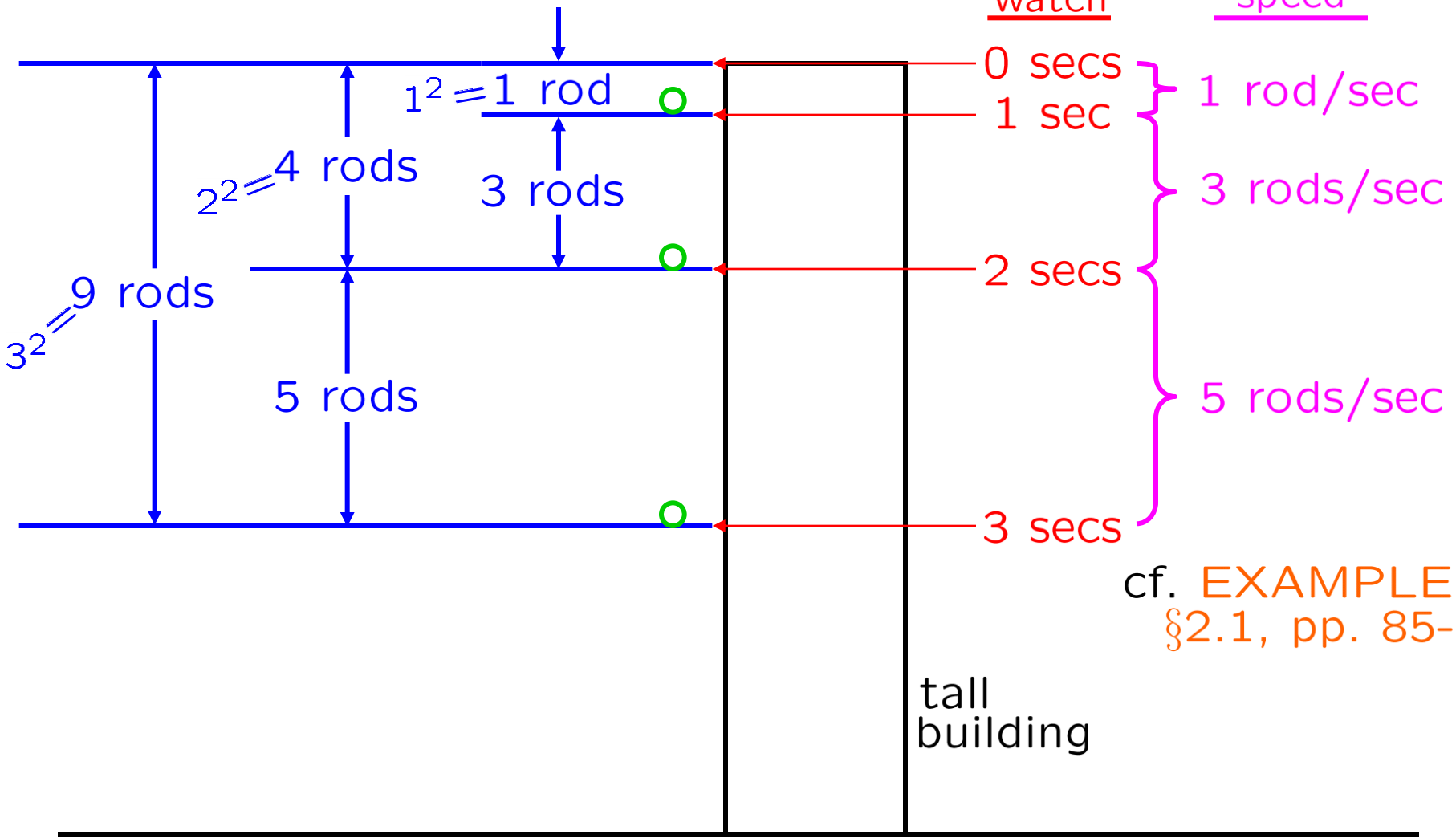
CALCULUS
Speed of a freely falling body

FACT: 1 rod $\approx$ 16 ft



cf. EXAMPLE 3
§2.1, pp. 85-86

FACT: 1 rod $\approx$ 16 ft



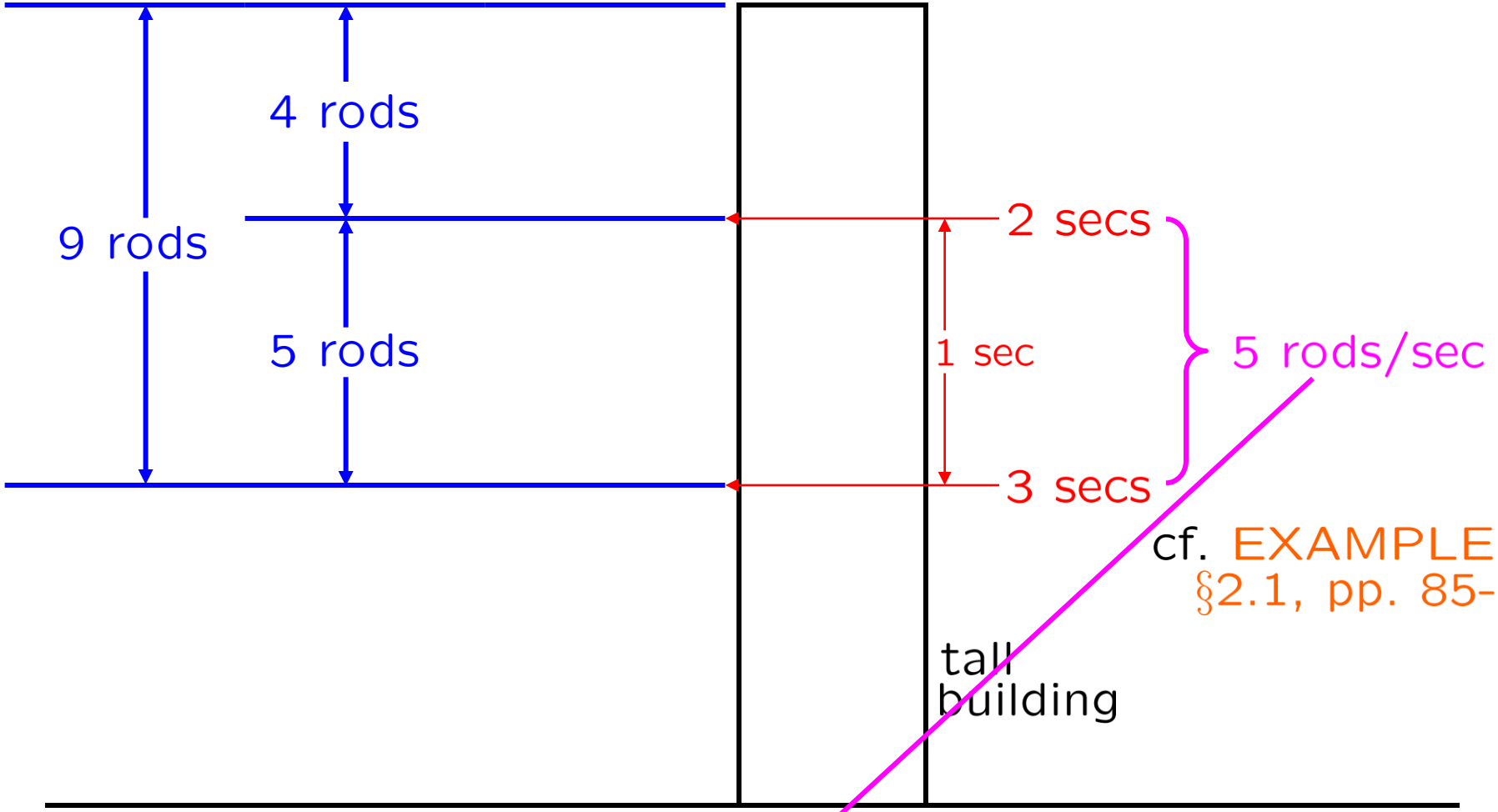
cf. EXAMPLE 3
§2.1, pp. 85-86

Question:
What is the speed at the 2 second mark?

FACT: 1 rod $\approx$ 16 ft

stop-watch

average speed



Approximate answer: 5 rods/sec

Better approximation by shortening the time interval from 1 sec to 0.5 secs

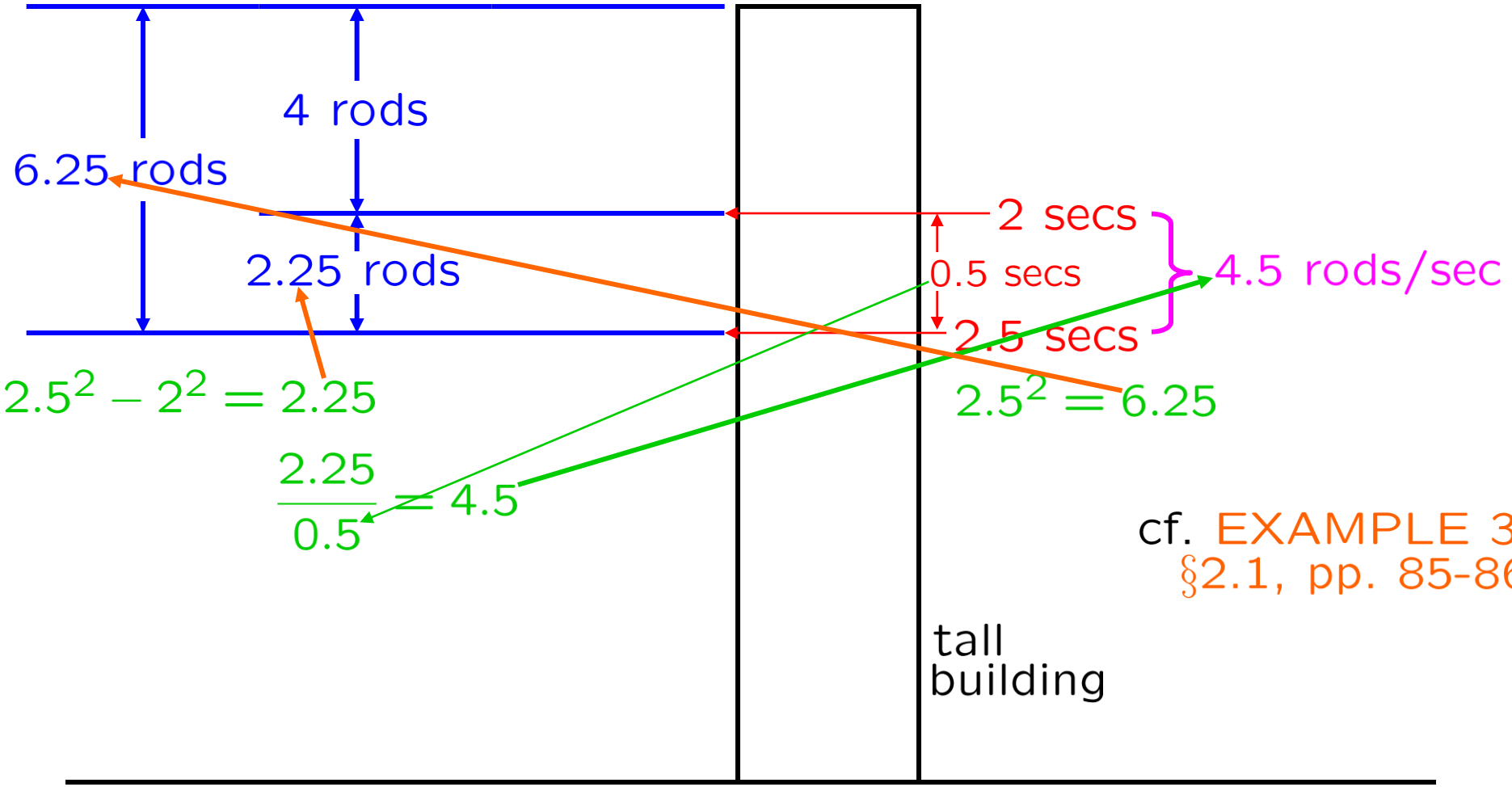
Question:

What is the speed at the 2 second mark?

FACT: 1 rod $\approx$ 16 ft

stop-watch

average speed



cf. EXAMPLE 3 §2.1, pp. 85-86

Approximate answer: 4.5 rods/sec

Better approximation by shortening the time interval from 0.5 sec to 0.1 secs

Question:

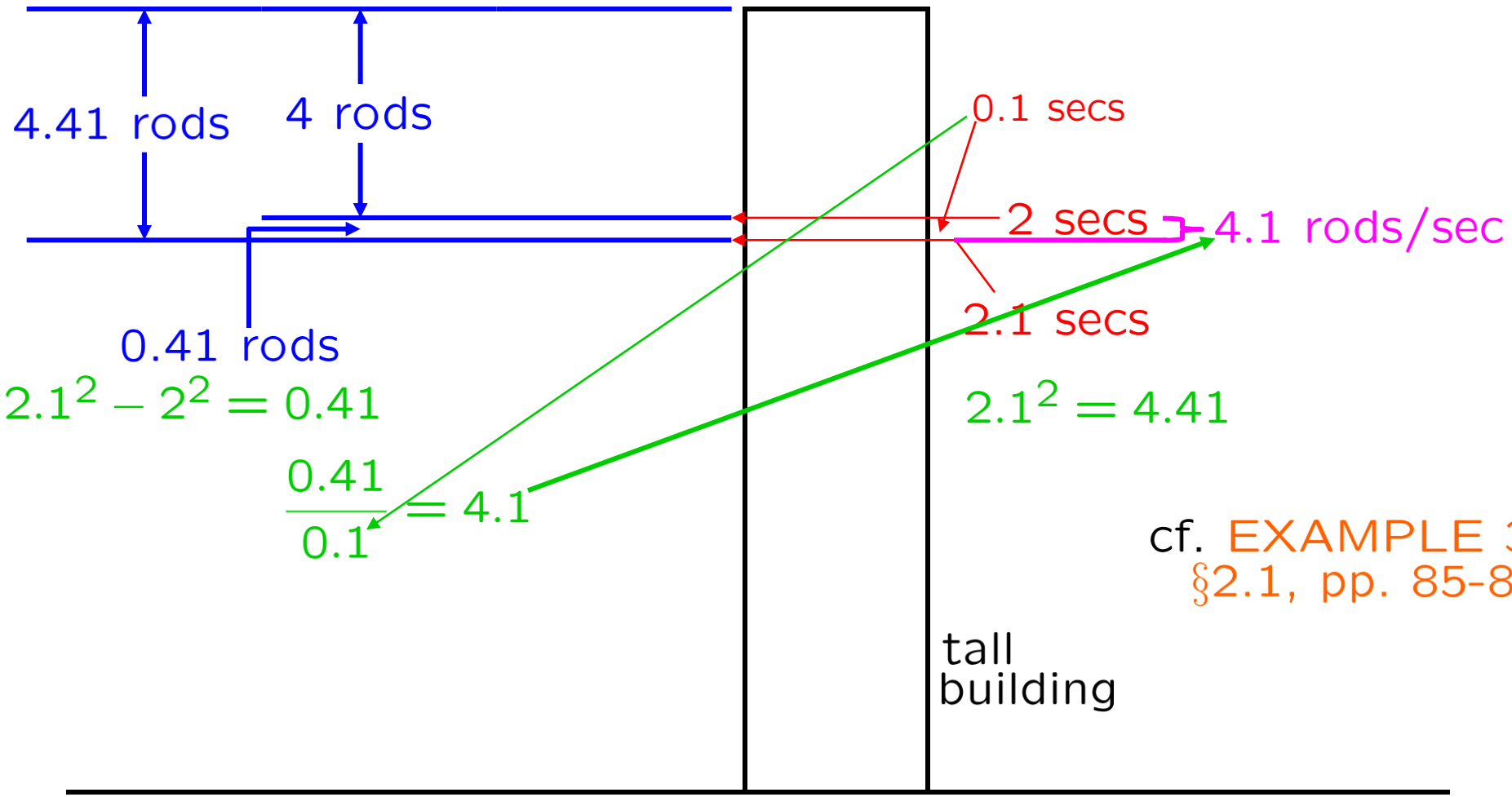
What is the speed at the 2 second mark?

§2.2

FACT: 1 rod $\approx$ 16 ft

stop-watch

average speed



cf. EXAMPLE 3 §2.1, pp. 85-86

Let's tabulate these answers . . .

Approximate answer: 4.1 rods/sec

Better approximation by shortening the time interval from 0.5 sec to 0.1 secs

Question:

What is the speed at the 2 second mark?

§2.2

End time	Start time	Time elapsed	End position	Start position	Distance traveled	Average speed
3	2	1	3^2	2^2	$3^2 - 2^2$	$\frac{3^2 - 2^2}{1} = 5$
2.5	2	0.5	2.5^2	2^2	$2.5^2 - 2^2$	$\frac{2.5^2 - 2^2}{0.5} = 4.5$
2.1	2	0.1	2.1^2	2^2	$2.1^2 - 2^2$	$\frac{2.1^2 - 2^2}{0.1} = 4.1$
2.01	2	0.01	2.01^2	2^2	$2.01^2 - 2^2$	$\frac{2.01^2 - 2^2}{0.01} = 4.01$
$2+h$	2	h	$(2+h)^2$	2^2	$(2+h)^2 - 2^2$	$\frac{(2+h)^2 - 2^2}{h}$

$$\frac{(2+h)^2 - 2^2}{h} = \frac{(2^2 + 4h + h^2) - 2^2}{h} = \frac{4h + h^2}{h} = \frac{(4+h)h}{h}$$

$$(2+h)^2 = 2^2 + 2(2)(h) + h^2 \qquad (a+b)^2 = a^2 + 2ab + b^2$$

End time	Start time	Time elapsed	End position	Start position	Distance traveled	Average speed
3	2	1	3^2	2^2	$3^2 - 2^2$	$\frac{3^2 - 2^2}{1} = 5$
2.5	2	0.5	2.5^2	2^2	$2.5^2 - 2^2$	$\frac{2.5^2 - 2^2}{0.5} = 4.5$
2.1	2	0.1	2.1^2	2^2	$2.1^2 - 2^2$	$\frac{2.1^2 - 2^2}{0.1} = 4.1$
2.01	2	0.01	2.01^2	2^2	$2.01^2 - 2^2$	$\frac{2.01^2 - 2^2}{0.01} = 4.01$
$2 + h$	2	h	$(2 + h)^2$	2^2	$(2 + h)^2 - 2^2$	$\frac{(2 + h)^2 - 2^2}{h}$

$$\frac{(2 + h)^2 - 2^2}{h} = \frac{(\cancel{2^2} + 4h + h^2) - \cancel{2^2}}{h} = \frac{4h + h^2}{h} = \frac{(4 + h)h}{h}$$

PROVIDED $h \neq 0$

$$= 4 + h$$

End time	Start time	Time elapsed	End position	Start position	Distance traveled	Average speed
3	2	1	3^2	2^2	$3^2 - 2^2$	$\frac{3^2 - 2^2}{1} = 5$
2.5	2	0.5	2.5^2	2^2	$2.5^2 - 2^2$	$\frac{2.5^2 - 2^2}{0.5} = 4.5$
2.1	2	0.1	2.1^2	2^2	$2.1^2 - 2^2$	$\frac{2.1^2 - 2^2}{0.1} = 4.1$
2.01	2	0.01	2.01^2	2^2	$2.01^2 - 2^2$	$\frac{2.01^2 - 2^2}{0.01} = 4.01$
$2 + h$	2	h	$(2 + h)^2$	2^2	$(2 + h)^2 - 2^2$	$4 + h$ PROVIDED $h \neq 0$

$$\frac{(2+h)^2 - 2^2}{h} = \frac{(\cancel{2^2} + 4h + h^2) - \cancel{2^2}}{h} = \frac{4h + h^2}{h} = \frac{(4+h)h}{h}$$

PROVIDED $h \neq 0$

$$= 4 + h$$

End time	Start time	Time elapsed	End position	Start position	Distance traveled	Average speed
	2	0.001	3^2	2^2		4.001
	2	0.0001	2.5^2	2^2		4.0001
	2	0.00001	2.1^2	2^2		4.00001
2.01	2	0.01	2.01^2	2^2	$2.01^2 - 2^2$	$\frac{2.01^2 - 2^2}{0.01} = 4.01$
$2+h$	2	h	$(2+h)^2$	2^2	$(2+h)^2 - 2^2$	$4+h$ PROVIDED $h \neq 0$

$$\frac{(2+h)^2 - 2^2}{h} = \frac{(2^2 + 4h + h^2) - 2^2}{h} = \frac{4h + h^2}{h} = \frac{(4+h)h}{h}$$

PROVIDED $h \neq 0$

$$= 4 + h$$

End time	Start time	Time elapsed	End position	Start position	Distance traveled	Average speed
	2	0.001	3^2	2^2		4.001
	2	0.0001	2.5^2	2^2		4.0001
	2	0.00001	2.1^2	2^2		4.00001
2.01	2	0.01	2.01^2	2^2	$2.01^2 - 2^2$	$\frac{2.01^2 - 2^2}{0.01} = 4.01$
$2+h$	2	h	$(2+h)^2$	2^2	$(2+h)^2 - 2^2$	$4+h$ PROVIDED $h \neq 0$

Approximate answers:

4.01, 4.001, 4.0001, 4.00001,. . . rods/sec

Exact answer: 4 rods/sec

End time	Start time	Time elapsed	End position	Start position	Distance traveled	Average speed
	2	0.001	3^2	2^2		4.001
	2	0.0001	2.5^2	2^2		4.0001
	2	0.00001	2.1^2	2^2		4.00001
2.01	2	0.01	2.01^2	2^2	$2.01^2 - 2^2$	$\frac{2.01^2 - 2^2}{0.01} = 4.01$
$2+h$	2	h	$(2+h)^2$	2^2	$(2+h)^2 - 2^2$	$4+h$ PROVIDED $h \neq 0$

Approximate answers: $\frac{(2+h)^2 - 2^2}{h} = 4 + h$ PROVIDED $h \neq 0$ rods/sec

Exact answer: $\lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} = 4$ rods/sec

$\neq$

SKILL

Avg & instantaneous speed

Whitman problems

§2.2, p. 26, #1-3

Approximate answers: $\dfrac{(2+h)^2-2^2}{h} = 4+h$ rods/sec PROVIDED $h \neq 0$

Exact answer: $\lim\limits_{h\to 0} \dfrac{(2+h)^2-2^2}{h} = 4$ rods/sec