CALCULUS
The derivative of a function is a function

The **derivative of a function** f is the function $\boxed{f'}$ defined by

$$f'(x) = \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$$

$$= \lim_{u \to x} \frac{[f(u)] - [f(x)]}{u - x}.$$

f' has all of the slopes of all of the tangent lines of f

e.g.: The slope of the tangent line of the graph of f at $(7, f(7))$ is: $f'(7)$

DEFINITION 2.18, §2.5, p. 42:

The **derivative of a function** f **at a number** a,

denoted by $\boxed{f'(a)}$, is $\displaystyle\lim_{h \to 0} \frac{[f(a+h)] - [f(a)]}{h}$

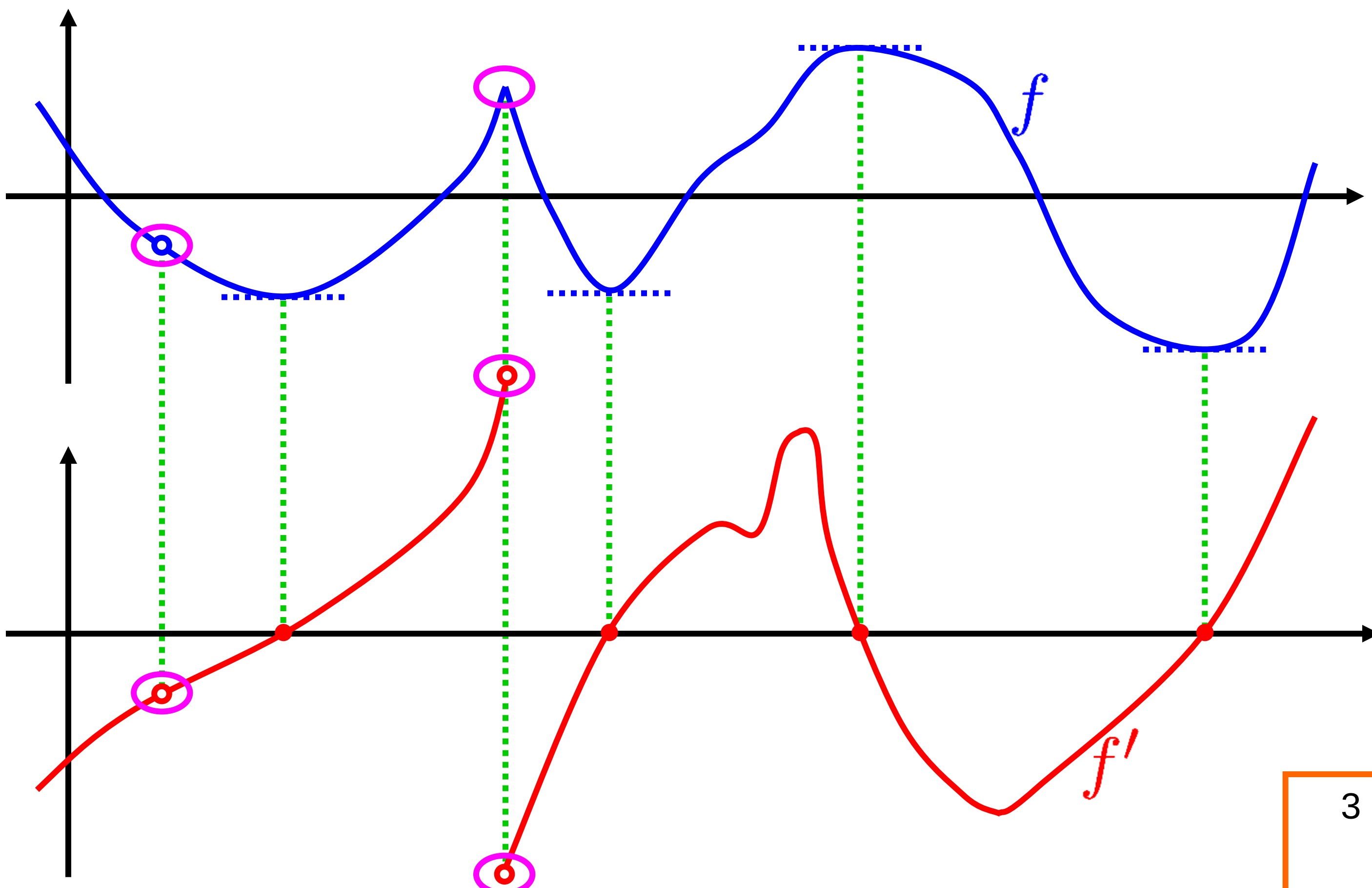
$$= \lim_{u \to a} \frac{[f(u)] - [f(a)]}{u - a}.$$

The **derivative of a function** f is the function $\boxed{f'}$ defined by

$$f'(x) = \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}.$$

Note: $\mathrm{dom}[f'] \subseteq \mathrm{dom}[f]$, but they may not be equal.

more about this when we compute the deriv. of $|\bullet|$



SKILL

gph f' from f

more about this skill later, when we study gphing

The **derivative of a function** f is the function $\boxed{f'}$ defined by

$$f'(x) = \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}.$$

Note: $\mathrm{dom}[f'] \subseteq \mathrm{dom}[f]$, but they may not be equal.

cf. DEFINITION 2.20, §2.5, p. 42:

We say f is **differentiable at** a if $a \in \mathrm{dom}[f']$,

i.e., if $f'(a)$ exists, *i.e.*, if $\lim_{h \to 0} \frac{[f(a+h)] - [f(a)]}{h}$ exists.

cf. DEFINITION 2.20, §2.5, p. 42:

We say f is **differentiable** if $\mathrm{dom}[f] = \mathrm{dom}[f']$,

i.e., if f is differentiable at every number in its domain.

The **derivative** of a function f is the function $\boxed{f'}$ defined by

$$f'(x) = \lim_{h\to 0} \frac{[f(x+h)] - [f(x)]}{h}.$$

e.g.: $f(x) = x^2$, $f'(x) = ??$

$$f(x+h) = (x+h)^2$$
$$= x^2 + 2xh + h^2$$

SUBTRACT $\quad f(x) = x^2$

$$[f(x+h)] - [f(x)] = 2xh + h^2$$

EXPR. OF x AND h

EXPR. OF x ALONE

$$\frac{[f(x+h)] - [f(x)]}{h} \underset{h\neq 0}{=} 2x + h \underset{h\to 0}{\to} 2x = f'(x)$$

"h" :→ "$chgtox$"? No, too long.

$chgtox \neq c \cdot h \cdot g \cdot t \cdot o \cdot x$

Old school: Replace "h" by "Δx" . . .

Old school: "$\triangle$" for "$chgto$"

The **derivative of a function** f

is the function $\boxed{f'}$ defined by

$$f'(x) = \lim_{\triangle x \to 0} \frac{[f(x + \triangle x)] - [f(x)]}{\triangle x}.$$

---

e.g.: $f(x) = x^2$, $f'(x) = ??$

$$f(x + \triangle x) = (x + \triangle x)^2$$
$$= x^2 + 2x(\triangle x) + (\triangle x)^2$$

SUBTRACT $\qquad f(x) = x^2$

$$[f(x + \triangle x)] - [f(x)] = 2x(\triangle x) + (\triangle x)^2$$

EXPR. OF x AND $\triangle x$

EXPR. OF x ALONE

$$\frac{[f(x + \triangle x)] - [f(x)]}{\triangle x} \underset{\triangle x \neq 0}{=} 2x + \triangle x \xrightarrow[\triangle x \to 0]{} 2x = f'(x) \blacksquare$$

"h" by "$chgtox$"? No, too long.

$chgtox \neq c \cdot h \cdot g \cdot t \cdot o \cdot x$

Old school: Replace "h" by "$\triangle x$" . . .

$\triangle x \neq \triangle \cdot x$

Old school: "$\triangle$" for "$chgto$"

The derivative of $(\bullet)^2$ is $2(\bullet)$, *i.e.*, $[(\bullet)^2]' = 2(\bullet)$.

The derivative of x^2 w.r.t. x is $2x$, *i.e.*, $\dfrac{d}{dx}[x^2] = 2x$.

---

e.g.: $f(x) = x^2$, $f'(x) = ??$

$$f(x + \triangle x) = (x + \triangle x)^2$$
$$= x^2 + 2x(\triangle x) + (\triangle x)^2$$

SUBTRACT $\qquad f(x) = x^2$

$$[f(x + \triangle x)] - [f(x)] = 2x(\triangle x) + (\triangle x)^2$$

$$\frac{[f(x + \triangle x)] - [f(x)]}{\triangle x} \underset{\triangle x \neq 0}{=} 2x + \triangle x \underset{\triangle x \to 0}{\to} 2x = f'(x) \blacksquare$$

"h" by "$chgtox$"? No, too long.

$chgtox \quad \neq \quad c \cdot h \cdot g \cdot t \cdot o \cdot x$

Old school: "$\triangle$" for "$chgto$"

Old school: Replace "h" by "$\triangle x$" . . .

$\triangle x \quad \neq \quad \triangle \cdot x$

The derivative of $(\bullet)^2$ is $2(\bullet)$, *i.e.*, $[(\bullet)^2]' = 2(\bullet)$.

The derivative of x^2 w.r.t. x is $2x$, *i.e.*, $\frac{d}{dx}[x^2] = 2x$.

e.g.: $y = x^2$, $\frac{dy}{dx} = ??$

$$[y]_{x:\to x+\triangle x} = (x + \triangle x)^2$$
$$= x^2 + 2x(\triangle x) + (\triangle x)^2$$
$$y = x^2$$

SUBTRACT

$$\triangle y = \underbrace{\left([y]_{x:\to x+\triangle x}\right) - y}_{\text{“}chgtoy\text{”}\,?} = 2x(\triangle x) + (\triangle x)^2$$

No, too long.

Old school:
"$\triangle$" for "*chgto*"

The derivative of $(\bullet)^2$ is $2(\bullet)$, *i.e.*, $[(\bullet)^2]' = 2(\bullet)$.

The derivative of x^2 w.r.t. x is $2x$, *i.e.*, $\frac{d}{dx}[x^2] = 2x$.

---

e.g.: $y = x^2$, $\frac{dy}{dx} = ??$

$$
\begin{aligned}
[y]_{x:\to x+\triangle x} &= (x+\triangle x)^2 \\
&= x^2 + 2x(\triangle x) + (\triangle x)^2 \\
y &= x^2
\end{aligned}
$$

SUBTRACT

$$\triangle y = \left([y]_{x:\to x+\triangle x}\right) - y = 2x(\triangle x) + (\triangle x)^2$$

$$\frac{y}{x} \neq \frac{\triangle y}{\triangle x} \underset{\triangle x \neq 0}{=} 2x + \triangle x$$

$$\frac{\triangle \cdot y}{\triangle \cdot x} \neq \frac{\triangle y}{\triangle x}$$

The derivative of $(\bullet)^2$ is $2(\bullet)$, *i.e.*, $[(\bullet)^2]' = 2(\bullet)$.

The derivative of x^2 w.r.t. x is $2x$, *i.e.*, $\frac{d}{dx}[x^2] = 2x$.

---

e.g.: $y = x^2$, $\frac{dy}{dx} = ??$

$$\begin{aligned} [y]_{x:\to x+\triangle x} &= (x+\triangle x)^2 \\ &= x^2 + 2x(\triangle x) + (\triangle x)^2 \\ y &= x^2 \end{aligned}$$

SUBTRACT

$$\triangle y = \left([y]_{x:\to x+\triangle x}\right) - y = 2x(\triangle x) + (\triangle x)^2$$

EXPR. OF x AND $\triangle x$ $\longrightarrow$ $\frac{\triangle y}{\triangle x} \underset{\triangle x \neq 0}{=} 2x + \triangle x \underset{\triangle x \to 0}{\to} 2x = \frac{dy}{dx}$ ■

NOTE: $\lim\limits_{\triangle x \to 0} \frac{\triangle y}{\triangle x} = \frac{dy}{dx}$ $\longleftarrow$ EXPR. OF x ALONE

§2.5

cf. §2.2, p. 25: Let $y = f(x)$.

Then $\boxed{\triangle y}$ is the traditional notation

for $[f(x + \triangle x)] - [f(x)]$,

i.e., $\triangle y = ([y]_{x:\to x+\triangle x}) - y$

which is an expression of

TWO variables: x and $\triangle x$.

Note: Some people are uncomfortable with a variable whose name involves two symbols, like "$\triangle x$", so most modern treatments of calculus replace "$\triangle x$" by "h", and avoid using "$\triangle y$".

NOT US!! . . .

cf. §2.2, p. 25: Let $y = f(x)$.

Then $\boxed{\triangle y}$ is the traditional notation

for $[f(x + \triangle x)] - [f(x)]$,

i.e., $\triangle y = ([y]_{x:\to x+\triangle x}) - y$

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TWO variables: x and $\triangle x$.

$$f'(x) = \lim_{h\to 0} \frac{[f(x+h)] - [f(x)]}{h}$$

$h :\to \triangle x$

$$f'(x) = \lim_{\triangle x\to 0} \frac{[f(x+\triangle x)] - [f(x)]}{\triangle x} = \lim_{\triangle x\to 0} \frac{\triangle y}{\triangle x}$$

an expression of x alone $\longrightarrow$ $\boxed{\frac{dy}{dx}}$ $:=$

same idea for expressions of t . . .

cf. §2.2, p. 25: Let $y = f(t)$.

Then $\boxed{\triangle y}$ is the traditional notation

for $[f(t + \triangle t)] - [f(t)]$,

i.e., $\triangle y = ([y]_{t:\to t+\triangle t}) - y$

which is an expression of

TWO variables: t and $\triangle t$.

$$f'(t) = \lim_{h\to 0} \frac{[f(t+h)] - [f(t)]}{h}$$

$$f'(t) = \lim_{\triangle t\to 0} \frac{[f(t+\triangle t)] - [f(t)]}{\triangle t} = \lim_{\triangle t\to 0} \frac{\triangle y}{\triangle t}$$

an expression of t alone $\longrightarrow \boxed{\dfrac{dy}{dt}} =:$

Same idea for any independent

and dependent variables. $y :\to r$

cf. §2.2, p. 25: Let $r = f(t)$.

Then $\boxed{\triangle r}$ is the traditional notation

for $[f(t + \triangle t)] - [f(t)]$,

i.e., $\triangle r = ([r]_{t:\to t+\triangle t}) - r$

which is an expression of

TWO variables: t and $\triangle t$.

$$f'(t) = \lim_{h\to 0} \frac{[f(t+h)] - [f(t)]}{h}$$

$$f'(t) = \lim_{\triangle t\to 0} \frac{[f(t+\triangle t)] - [f(t)]}{\triangle t} = \lim_{\triangle t \to 0} \frac{\triangle r}{\triangle t}$$

an expression of t alone $\longrightarrow$ $\boxed{\dfrac{dr}{dt}} :=$

Next: examples

Same idea for any independent and dependent variables.

Let $r := \sqrt{t}$.

Goal: $\dfrac{dr}{dt} = \dfrac{1}{2\sqrt{t}}$ ■

SKILL
Find deriv from the def'n

CALCULUS is all about learning to do calculations like this quickly and algorithmically.

$$\Delta r = \sqrt{t+\Delta t} - \sqrt{t}$$

$$= \frac{\sqrt{t+\Delta t} - \sqrt{t}}{1} \; \frac{\sqrt{t+\Delta t} + \sqrt{t}}{\sqrt{t+\Delta t} + \sqrt{t}}$$

$$= \frac{(\cancel{t} + \Delta t) - \cancel{t}}{\sqrt{t+\Delta t} + \sqrt{t}}$$

$$= \frac{\Delta t}{\sqrt{t+\Delta t} + \sqrt{t}}$$

$$\frac{d}{dt}[\sqrt{t}] = \frac{1}{2\sqrt{t}}$$

$$\frac{\Delta r}{\Delta t} \underset{\Delta t \neq 0}{=} \frac{1}{\sqrt{t+\boxed{\Delta t}} + \sqrt{t}} \xrightarrow[\Delta t \to 0]{} \frac{1}{\sqrt{t} + \sqrt{t}} = \frac{1}{2\sqrt{t}}$$

0

Back to x and y...

§2.2

SKILL
Find deriv from the def'n

Let $y := \sqrt{x}$. Goal: $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$ ■

CALCULUS is all about learning to do calculations like this quickly and algorithmically.

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

$$= \frac{\sqrt{x+\Delta x} - \sqrt{x}}{1} \; \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \frac{(\cancel{x} + \Delta x) - \cancel{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \frac{\Delta x}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\frac{\Delta y}{\Delta x} \underset{\Delta x \neq 0}{=} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} \xrightarrow[\Delta x \to 0]{} \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$

$$[\sqrt{\bullet}]' = \frac{1}{2\sqrt{\bullet}}$$

Let $y := \dfrac{1}{x}$.

SKILL

Find deriv from the def'n

$$\triangle y = ([y]_{x:\to x+\triangle x}) - y$$

$$= \frac{1}{x+\triangle x} - \frac{1}{x}$$

$$= \frac{\cancel{x} - (\cancel{x} + \triangle x)}{(x+\triangle x)x}$$

$$= \frac{-\triangle x}{(x+\triangle x)x}$$

DIVIDE BY $\triangle x$

Goal: $\dfrac{dy}{dx} = \dfrac{-1}{(x+0)x} = -\dfrac{1}{x^2}$

$$\frac{\triangle y}{\triangle x} \underset{\triangle x \neq 0}{=} \frac{-1}{(x+\triangle x)x}$$

Let $\triangle x \to 0$.

Goal: Equation of the tangent line to $y = 1/x$ at $(2, 1/2)$.

WRONG: $y - \dfrac{1}{2} = -\dfrac{1}{x^2}(x-2)$

$-\dfrac{1}{x^2}$ has all slopes of all tangent lines.

We need ONE slope of ONE tangent line.

SKILL

find slope of tan line

Let $y := \frac{1}{x}$.

SKILL
Find deriv from the def'n

$$\triangle y = ([y]_{x :\to x+\triangle x}) - y$$

$$= \frac{1}{x+\triangle x} - \frac{1}{x}$$

$$= \frac{\cancel{x} - (\cancel{x} + \triangle x)}{(x+\triangle x)x}$$

$$= \frac{-\triangle x}{(x+\triangle x)x}$$

$$\frac{\triangle y}{\triangle x} \underset{\triangle x \neq 0}{=} \frac{-1}{(x+\triangle x)x}$$

Let $\triangle x \to 0$.

Goal: $\frac{dy}{dx} = \frac{-1}{(x+0)x} = -\frac{1}{x^2}$

SKILL
find slope of tan line

Goal: Equation of the tangent line to $y = 1/x$ at $(2, 1/2)$.

$$\text{Slope} = [dy/dx]_{x :\to 2}$$
$$= [-1/x^2]_{x :\to 2}$$
$$= -1/4$$

SKILL
pt/slope to eqn

Equation:
$$y = (1/2) + (-1/4)(x-2)$$

SKILL
find eq'n of tan line

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \frac{[f(x+h)]-[f(x)]}{h}$

Alternate notation: $f'(x) = \lim_{u\to x} \frac{[f(u)]-[f(x)]}{u-x}$

Let $y := f(x)$

$$\cancel{[f(x)]'}$$
$$/\!/$$

$$f'(x) = \underline{\cancel{y'}} = \frac{dy}{dx} = \cancel{\frac{df}{dx}} = \frac{d}{dx}[f(x)]$$

exception for
"implicit differentiation"
in §4.9

For *these* slides, remember:

$$f'(x) = \frac{dy}{dx} = \frac{d}{dx}[f(x)]$$

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \dfrac{[f(x+h)] - [f(x)]}{h}$

cf. §2.5, p. 42, DEFINITION 2.20:

A function f is **differentiable at** a if $f'(a)$ exists.

It is **differentiable on an open interval**

(p, q) or (p, ∞) or $(-\infty, q)$ or $(-\infty, \infty)$

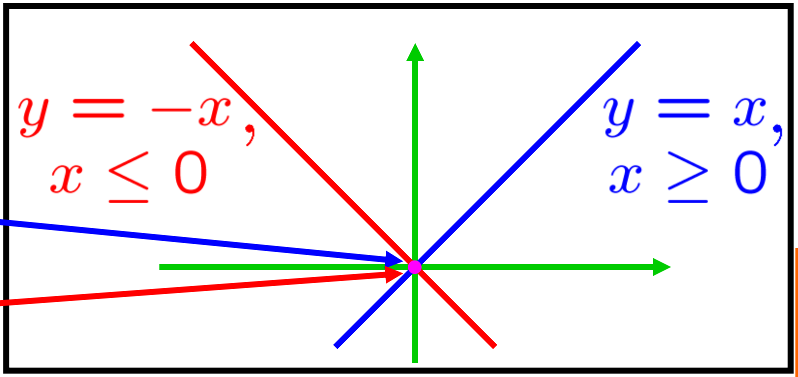
if it is differentiable at every number in the interval.

EXAMPLE: Where is the function $f(x) = |x|$ differentiable?

Understood: f is the function, *NOT* $f(x)$

$f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$



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It is **differentiable on an open interval**

(p, q) or (p, ∞) or $(-\infty, q)$ or $(-\infty, \infty)$

if it is differentiable at every number in the interval.

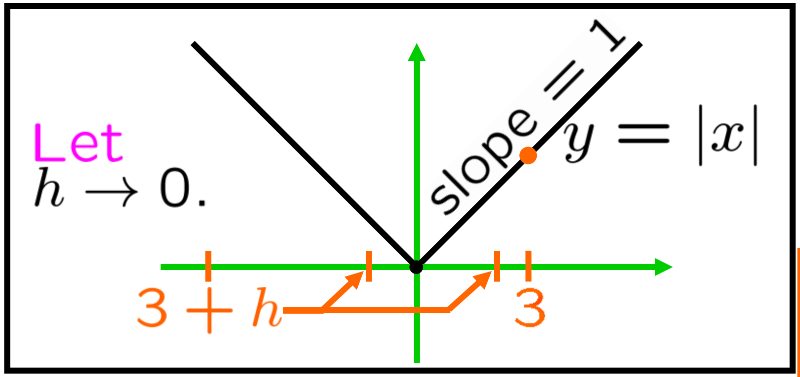
EXAMPLE: Where is the function $f(x) = |x|$ differentiable?

$$f'(3) = \lim_{h \to 0} \frac{\boxed{[f(3+h)]} - \boxed{[f(3)]}}{\boxed{h}} \overset{??}{\underset{\text{equal}}{=}} \lim_{h \to 0} \frac{\boxed{[3+h]} - \boxed{[3]}}{\boxed{h}}$$

equal for h sufficiently close to 0

NOT equal

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$



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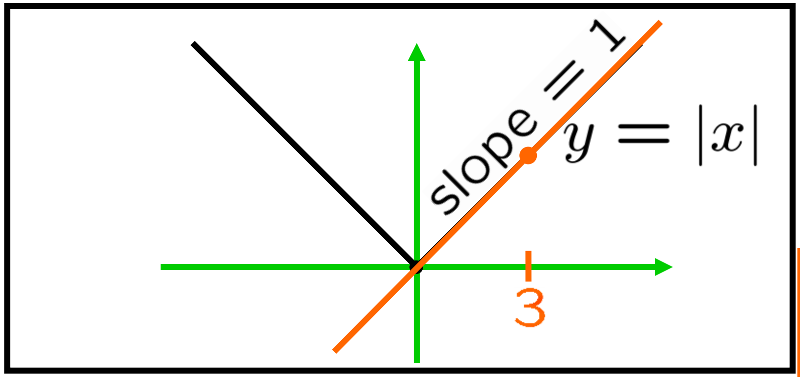
if it is differentiable at every number in the interval.

EXAMPLE: Where is the function $f(x)=|x|$ differentiable?

$$f'(3) = \lim_{h\to 0} \boxed{\frac{[f(3+h)]-[f(3)]}{h}} \overset{??}{=} \lim_{h\to 0} \boxed{\frac{[\cancel{3}+h]-[\cancel{3}]}{h}} = 1$$

$3 :\to x$, for $x>0$ equal for h sufficiently close to 0

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$



§2.4

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \frac{[f(x+h)] - [f(x)]}{h}$

cf. §2.5, p. 42, DEFINITION 2.20:

A function f is **differentiable at** a if $f'(a)$ exists.

It is **differentiable on an open interval**

(p,q) or (p,∞) or $(-\infty,q)$ or $(-\infty,\infty)$

if it is differentiable at every number in the interval.

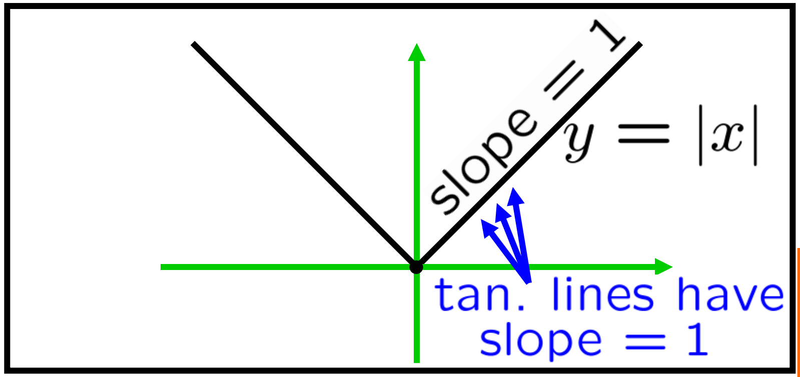
EXAMPLE: Where is the function $f(x) = |x|$ differentiable?

$\forall x > 0$,

$$f'(x) = \lim_{h\to 0} \frac{[f(x+h)] - [f(x)]}{h} = \lim_{h\to 0} \frac{[\cancel{x} + h] - [\cancel{x}]}{h} = 1$$

f is differentiable on $(0,\infty)$.

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$



$$\text{Recall: } f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \frac{[f(x+h)] - [f(x)]}{h}$$

cf. §2.5, p. 42, DEFINITION 2.20:

A function f is **differentiable at** a if $f'(a)$ exists.

It is **differentiable on an open interval**

$$(p,q) \quad \text{or} \quad (p,\infty) \quad \text{or} \quad (-\infty,q) \quad \text{or} \quad (-\infty,\infty)$$

if it is differentiable at every number in the interval.

EXAMPLE: Where is the function $f(x) = |x|$ differentiable?

$\forall x < 0$,

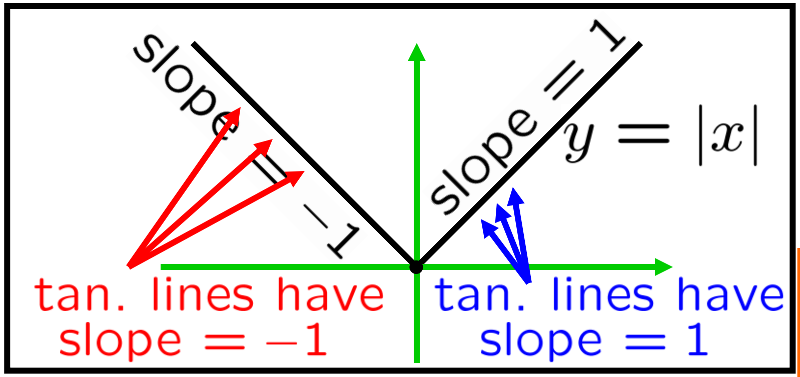
$$f'(x) = \lim_{h\to 0} \frac{[f(x+h)] - [f(x)]}{h} = \lim_{h\to 0} \frac{[-(\cancel{x}+h)] - [-\cancel{x}]}{h} = -1$$

at $x = 0$?

f is differentiable on $(-\infty, 0)$.

f is differentiable on $(0, \infty)$.

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$



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Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \frac{[f(x+h)]-[f(x)]}{h}$

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(p,q) or (p,∞) or $(-\infty,q)$ or $(-\infty,\infty)$

if it is differentiable at every number in the interval.

EXAMPLE: Where is the function $f(x)=|x|$ differentiable?

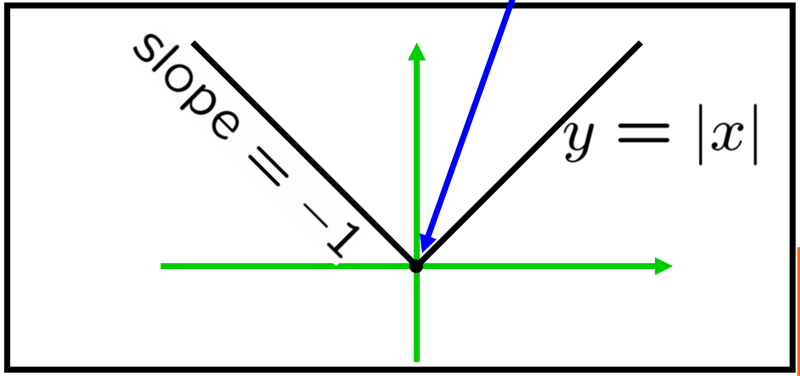
$$\lim_{h\to 0\boxed{+}} \frac{[f(0+h)]-[f(0)]}{h} = \lim_{h\to 0+} \frac{h}{h} = 1$$

at $x=0$?

right tan. line has slope $=1$

f is differentiable on $(-\infty,0)$.

f is differentiable on $(0,\infty)$.



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if it is differentiable at every number in the interval.

EXAMPLE: Where is the function $f(x) = |x|$ differentiable?

$$\lim_{h \to 0^{\boxed{-}}} \frac{[f(0+h)] - [f(0)]}{h} = \lim_{h \to 0^-} \frac{-h}{h} = -1$$

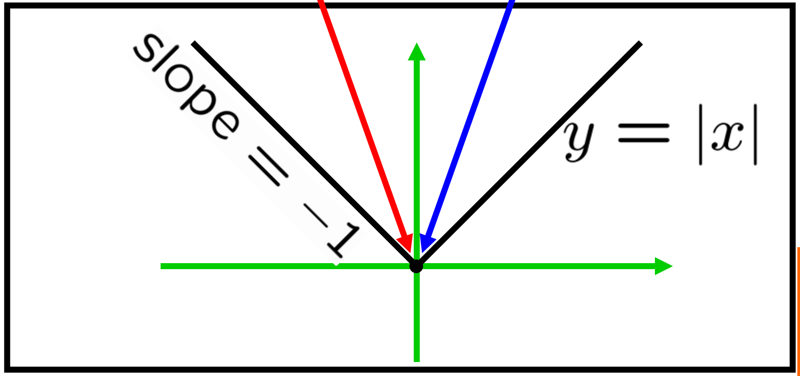
at $x = 0$?

left tan. line has slope $= -1$

right tan. line has slope $= 1$

f is differentiable on $(-\infty, 0)$.

f is differentiable on $(0, \infty)$.



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(p,q) or (p,∞) or $(-\infty,q)$ or $(-\infty,\infty)$

if it is differentiable at every number in the interval.

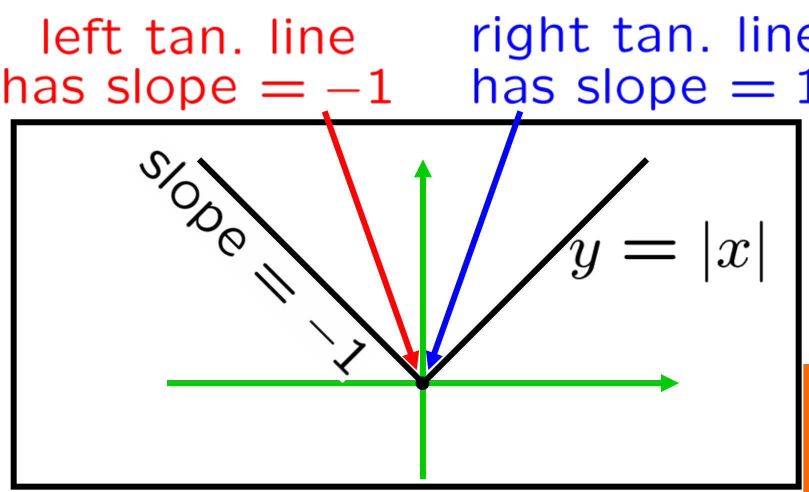
EXAMPLE: Where is the function $f(x) = |x|$ differentiable?

$$\lim_{h\to 0} \frac{[f(0+h)] - [f(0)]}{h} \text{ DNE}$$

at $x = 0$?

f is differentiable on $(-\infty, 0)$.
f is differentiable on $(0, \infty)$.

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$



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cf. §2.5, p. 42, DEFINITION 2.20:

A function f is **differentiable at** a if $f'(a)$ exists.

It is **differentiable on an open interval**

(p,q) or (p,∞) or $(-\infty,q)$ or $(-\infty,\infty)$

if it is differentiable at every number in the interval.

EXAMPLE: Where is the function $f(x) = |x|$ differentiable?

$\lim_{h\to 0} \frac{[f(0+h)]-[f(0)]}{h}$ DNE

So: contin. at $a \not\Rightarrow$ diff. at a, but. . .

at $x = 0$?

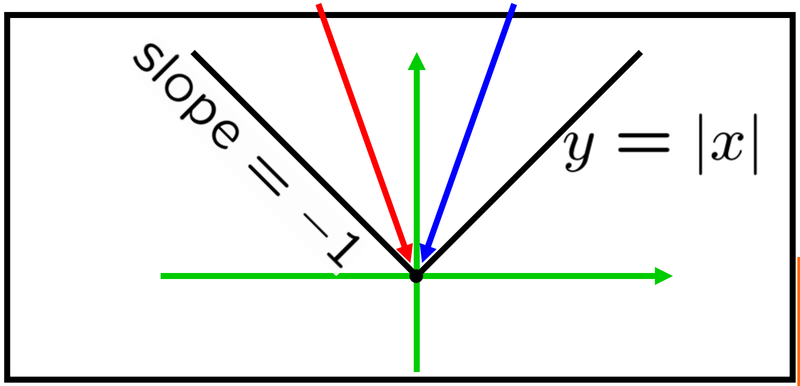
f is not differentiable at 0.

f is differentiable on $(-\infty, 0)$.

f is differentiable on $(0,\infty)$.

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$

"two-sided" tan. line DNE



§2.4

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \dfrac{[f(x+h)]-[f(x)]}{h}$

cf. §2.5, p. 42, DEFINITION 2.20:

A function f is **differentiable at** a if $f'(a)$ exists.

It is **differentiable on an open interval**

$$(p,q) \quad \text{or} \quad (p,\infty) \quad \text{or} \quad (-\infty,q) \quad \text{or} \quad (-\infty,\infty)$$

if it is differentiable at every number in the interval.

TH'M: If f is differentiable at a, then f is continuous at a.

Pf: $\dfrac{(f(\boxed{u}))-(f(a))}{\boxed{u}-a} \to f'(a)$, $\quad u :\to x \quad$ as $\boxed{u} \to a$

So: contin. at $a \not\Rightarrow$ diff. at a, but...

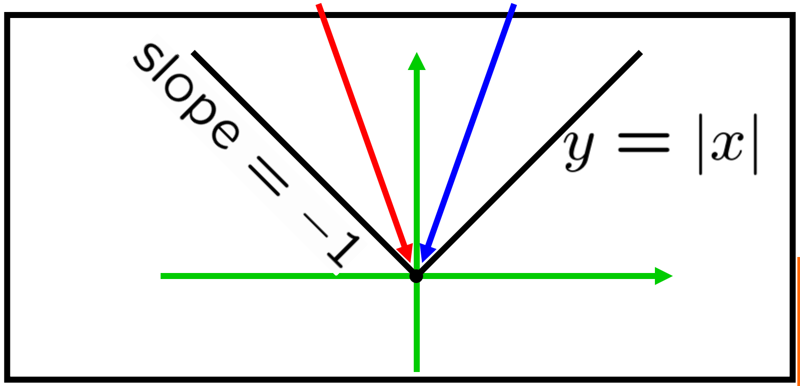
f is not differentiable at 0.

f is differentiable on $(-\infty,0)$.

f is differentiable on $(0,\infty)$.

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$

"two-sided" tan. line DNE



Spp

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$

cf. §2.5, p. 42, DEFINITION 2.20:

A function f is **differentiable at** a if $f'(a)$ exists.

It is **differentiable on an open interval**

$$(p,q) \quad \text{or} \quad (p,\infty) \quad \text{or} \quad (-\infty,q) \quad \text{or} \quad (-\infty,\infty)$$

if it is differentiable at every number in the interval.

TH'M: If f is differentiable at a, then f is continuous at a.

Pf: $\frac{(f(\boxed{x})) - (f(a))}{\boxed{x} - a} \to f'(a)$, as $\boxed{x} \to a$

So: contin. at $a \not\Rightarrow$ diff. at a, but...

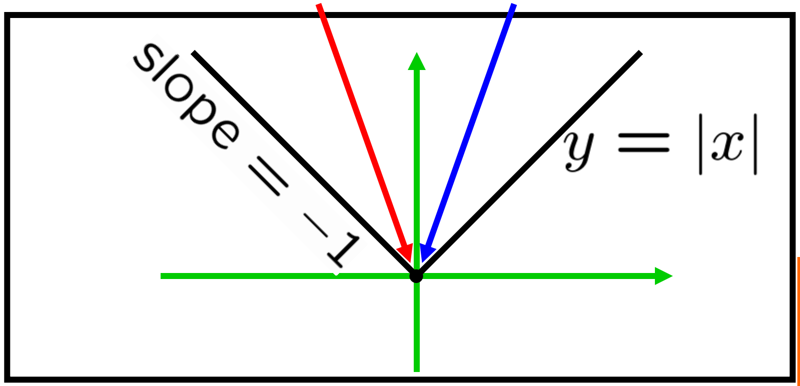
"two-sided" tan. line DNE

f is not differentiable at 0.

f is differentiable on $(-\infty, 0)$.

f is differentiable on $(0, \infty)$.

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$



Spp

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if it is differentiable at every number in the interval.

TH'M: If f is differentiable at a, then f is continuous at a.

Pf:

$$\left.\begin{array}{ll} \dfrac{(f(x))-(f(a))}{x-a} \to f'(a), & \text{as } x \to a \\ x-a \to 0, & \text{as } x \to a \end{array}\right\} \text{MULTIPLY TOGETHER}$$

$$\left[\frac{(f(x))-(f(a))}{\cancel{x-a}}\right][\cancel{x-a}] \to [f'(a)][0], \text{ as } x \to a$$

$$\|x \neq a \qquad\qquad \| \qquad 0$$

$$(f(x)) - (f(a))$$

Spp

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \dfrac{[f(x+h)] - [f(x)]}{h}$

cf. §2.5, p. 42, DEFINITION 2.20:

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if it is differentiable at every number in the interval.

TH'M: If f is differentiable at a, then f is continuous at a.

Pf: $(f(x)) - (f(a)) \to 0, \qquad$ as $x \to a$

$f(a) \;\to f(a),$ as $x \to a$

$\to \qquad\qquad$, as $x \to a$

0

$(f(x)) - (f(a))$

Spp

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \dfrac{[f(x+h)] - [f(x)]}{h}$

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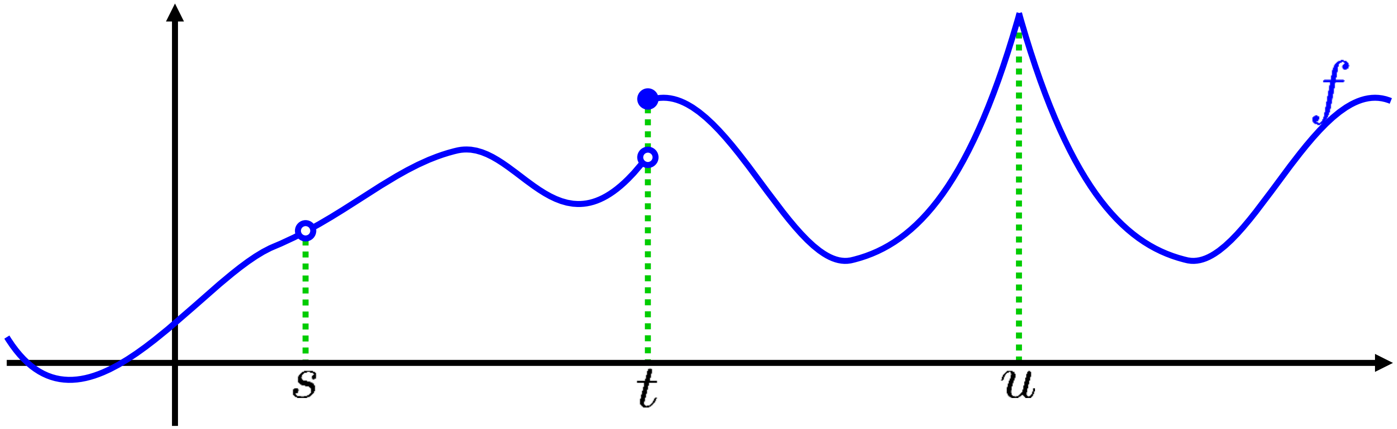
if it is differentiable at every number in the interval.

TH'M: If f is differentiable at a, then f is continuous at a.

$$\text{Pf:} \quad \left.\begin{aligned} (f(x)) - (f(a)) &\to 0, && \text{as } x \to a \\ f(a) &\to f(a), && \text{as } x \to a \end{aligned}\right\} \text{ADD TOGETHER}$$

$$f(x) \to f(a), \text{ as } x \to a$$

$$\lim_{x \to a} (f(x)) = f(a) \quad \text{QED}$$



f not def'd at s, so not contin. at s, so not diff. at s

f not contin. at t, so not diff. at t

f not diff. at u

TH'M: If f is differentiable at a, then f is continuous at a.

Pf: $(f(x)) - (f(a)) \to 0$, as $x \to a$

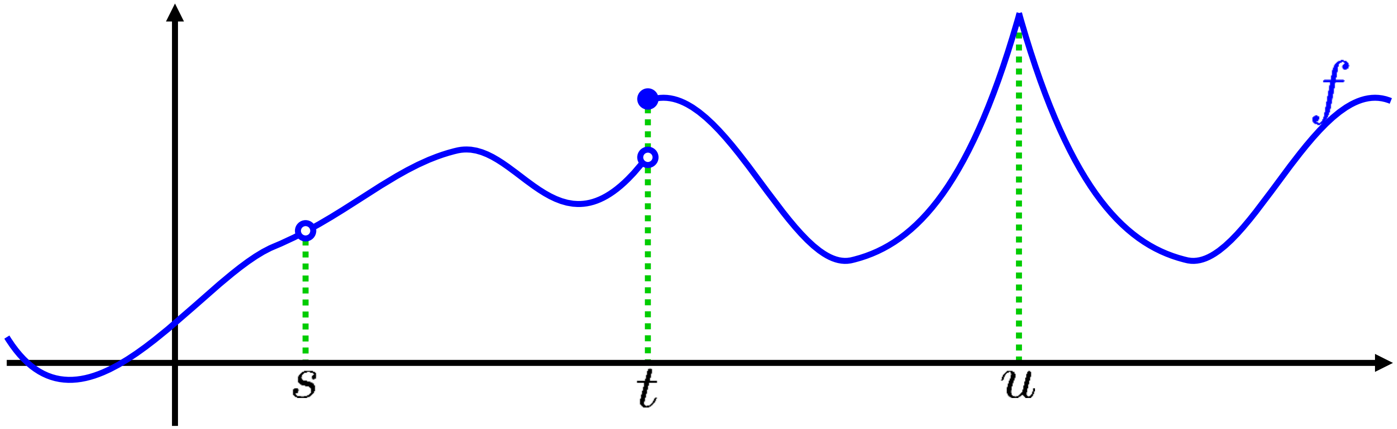
$f(a) \to f(a)$, as $x \to a$

ADD TOGETHER

$f(x) \to f(a)$, as $x \to a$

$\lim_{x \to a} (f(x)) = f(a)$ QED

Spp



f not def'd at s, so not contin. at s, so not diff. at s

f not contin. at t, so not diff. at t

f not diff. at u

"defined" : There's a dot there.

"continuous" : There's no break there.

"differentiable" : There's neither a break nor a sudden change of direction there.