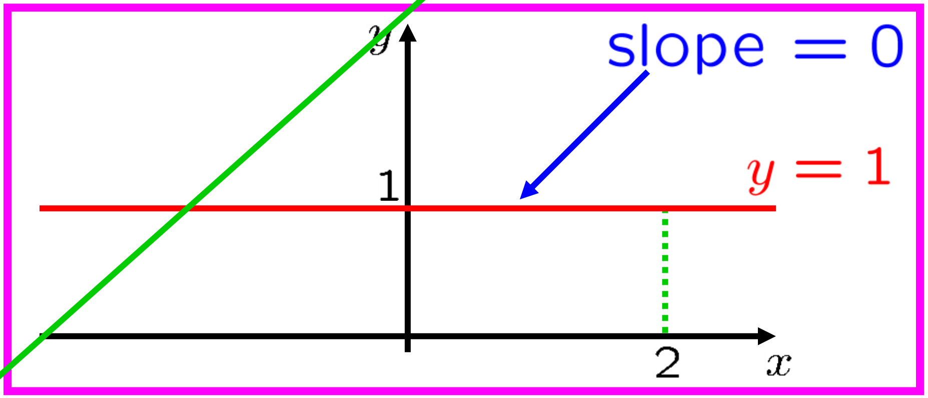
CALCULUS
The power rule

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$

e.g.:

Predict: $f'(2) = 0$

Let $f(x) = 1$.



any tangent line to a line is the line itself

$$\frac{[f(x+h)] - [f(x)]}{h} = \frac{1-1}{h} \overset{h \neq 0}{=} 0 \to 0, \text{ as } h \to 0$$

$$\frac{d}{dx}[1] = f'(x) = 0$$

$$1 :\to c$$

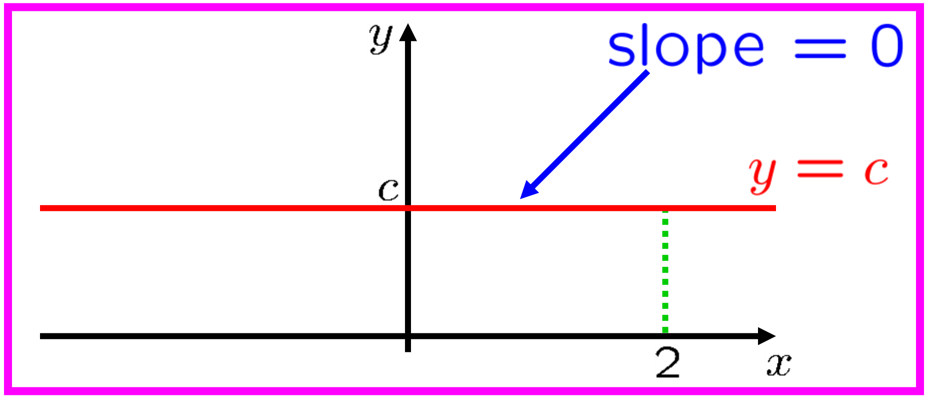
Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \frac{[f(x+h)] - [f(x)]}{h}$

Predict: $f'(2) = 0$

e.g.:

Let $c \in \mathbb{R}$.

Let $f(x) = c$.



any tangent line to a line is the line itself

$$\frac{[f(x+h)] - [f(x)]}{h} = \frac{c-c}{h} \overset{h\neq 0}{=} 0 \to 0, \text{ as } h \to 0$$

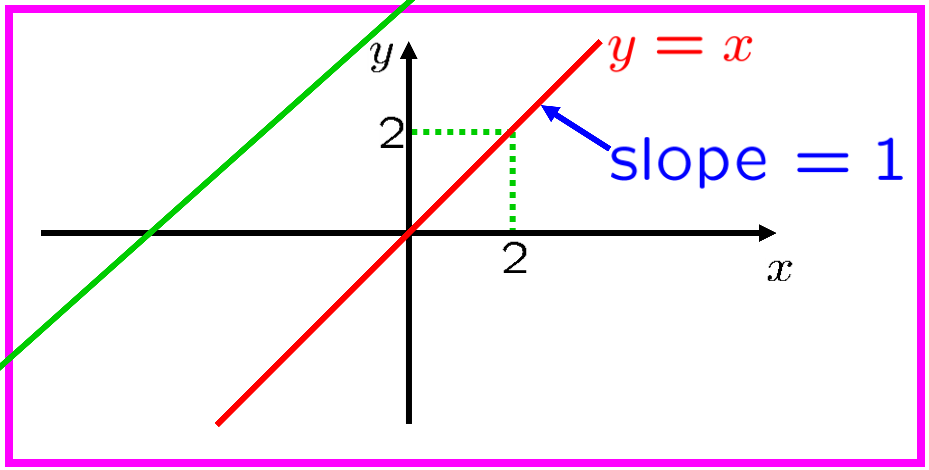
$$\frac{d}{dx}[c] = f'(x) = 0$$

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$

Predict: $f'(2) = 1$

e.g.:

Let $f(x) = x$.



any tangent line to a line is the line itself

$$\frac{[f(x+h)] - [f(x)]}{h} = \frac{[x + h] - [x]}{h} \overset{h \neq 0}{=} 1 \to 1, \text{ as } h \to 0$$

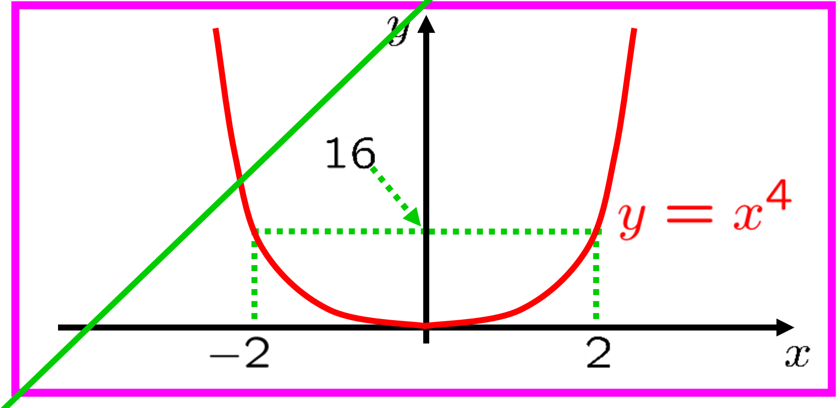
$$\frac{d}{dx}[x] = f'(x) = 1$$

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$

e.g.:

Predict: $f'(2) > 0$

Let $f(x) = x^4$.



$f'(-2) < 0$

$f'(0) = 0$

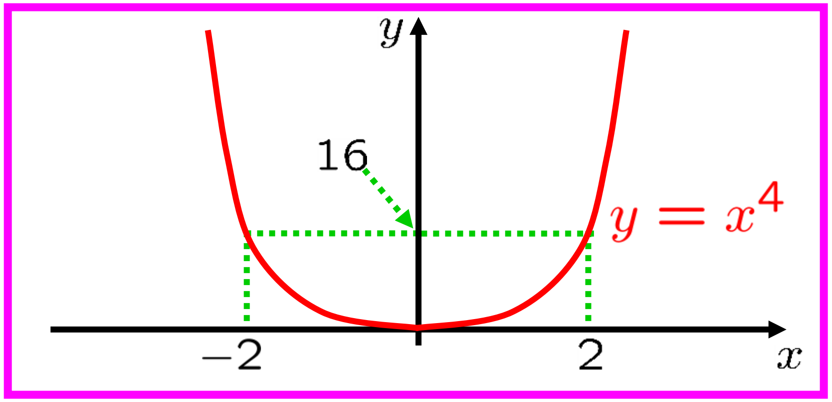
$$(x+h)^4 = 1\,x^4 + 4\,x^3h + 6\,x^2h^2 + 4\,xh^3 + 1\,h^4$$

$$\frac{(x+h)^4 - x^4}{h}$$

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \frac{[f(x+h)] - [f(x)]}{h}$

e.g.: Predict: $f'(2) > 0$ ☺

Let $f(x) = x^4$.



$f'(-2) < 0$ ☺

$f'(0) = 0$ ☺

$(x+h)^4 = 1x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + 1h^4$

$(x+h)^4 - x^4 = 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

$\frac{(x+h)^4 - x^4}{h} \overset{h\neq 0}{=} 4x^3 + 6x^2h + 4xh^2 + h^3$ $\xrightarrow{h \to 0}$ 0

$\to 4x^3$, as $h \to 0$

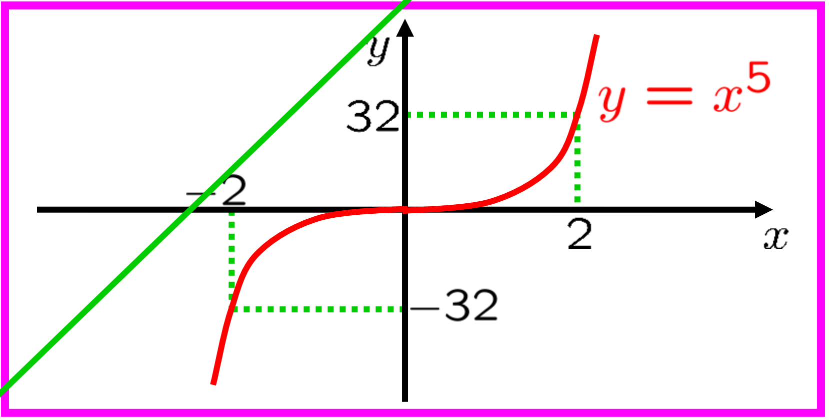
$\frac{d}{dx}[x^4] = f'(x) = 4x^3$

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$

e.g.:

Predict: $f'(2) > 0$

Let $f(x) = x^5$.



$f'(-2) > 0$

$f'(0) = 0$

$$(x+h)^5 = 1x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + 1h^5$$

$$\frac{(x+h)^5 - x^5}{h}$$

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$

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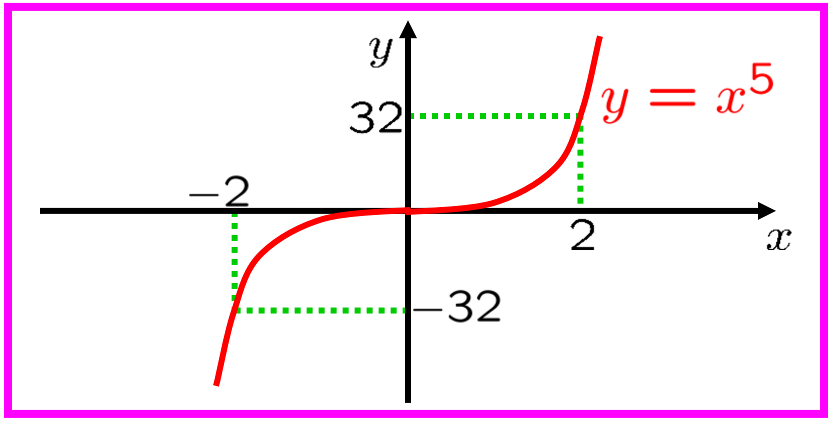

$f'(-2) > 0$

$f'(0) = 0$

$$(x+h)^5 = 1x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + 1h^5$$

$$(x+h)^5 - x^5 = 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$

$$\frac{(x+h)^5 - x^5}{h} \overset{h \neq 0}{=} 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$$

$$\to 5x^4, \qquad \text{as } h \to 0$$

$$\frac{d}{dx}[x^5] = f'(x) = 5x^4$$

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$

e.g.: $\frac{d}{dx}[x^5] = 5x^4$

$\frac{d}{dx}[x^4] = 4x^3$

$\frac{d}{dx}[x^5] = \qquad 5x^4$

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \frac{[f(x+h)]-[f(x)]}{h}$

e.g.: $\frac{d}{dx}[x^5] = 5x^4$

$\frac{d}{dx}[x^4] = 4x^3$

True for $n = 4, 5$.

$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}} = \frac{1}{2}\frac{1}{x^{1/2}} = \frac{1}{2}x^{-1/2}$

$\frac{d}{dx}[x^{1/2}] = \frac{1}{2}x^{(1/2)-1}$

True for $n = 1/2$.

cf. §3.1, p. 74

THE POWER RULE:

$\forall$integers $n > 1$,

$\frac{d}{dx}[x^n] = nx^{n-1}$

$\frac{d}{dx}\left[\frac{1}{x}\right] = \frac{-1}{x^2} = (-1)\frac{1}{x^2} = (-1)x^{-2}$

$\frac{d}{dx}[x^{-1}] = (-1)x^{-1-1}$

True for $n = -1$.

Not quite true for $n = 1$:

$\frac{d}{dx}[x^1] = \frac{d}{dx}[x] = 1 \overset{x\neq 0}{=} 1x^{1-1}$

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \frac{[f(x+h)] - [f(x)]}{h}$

e.g.: $\frac{d}{dx}[x^5] = 5x^4$

$\frac{d}{dx}[x^4] = 4x^3$

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$\frac{d}{dx}[x^n] = nx^{n-1}$

$\frac{d}{dx}\left[\frac{1}{\sqrt{x}}\right] = \frac{-1}{2x\sqrt{x}}$

$\frac{d}{dx}[x^{-1}] = (-1)x^{-1-1}$

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Not quite true for $n = 1$:

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$\frac{d}{dx}[x^1] = \frac{d}{dx}[x] = 1$

$\|_{x \neq 0}$

$1x^{1-1}$

$\frac{d}{dx}\left[\frac{1}{\sqrt{x}}\right] = \frac{-1}{2x\sqrt{x}} = -\frac{1}{2}\frac{1}{x}\frac{1}{x^{1/2}}$

$= -\frac{1}{2}x^{-1}x^{-1/2}$

$\frac{d}{dx}[x^{-1/2}] = -\frac{1}{2}x^{(-1/2)-1}$

True for $n = -1/2$.

§3.1

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$

e.g.: $\frac{d}{dx}[x^5] = 5x^4$

$\frac{d}{dx}[x^4] = 4x^3$

True for $n = 4, 5$.

True for $n = 1/2, -1$.

True for $n = -1/2$.

True for $n = 1/2, -1$.

cf. §3.1, p. 74

THE POWER RULE:

$\forall$integers $n > 1$,

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

cf. §3.1, p. 74

THE POWER RULE:
(GENERAL VERSION):

$\forall n \in \mathbb{R}$,

$$\frac{d}{dx}[x^n] \underset{x \neq 0}{=} nx^{n-1}$$

Not quite true for $n = 1$:

$$\frac{d}{dx}[x^1] = \frac{d}{dx}[x] = 1 \underset{x \neq 0}{=} 1x^{1-1}$$

True for $n = -1/2$.

§3.1

Recall: $f'(x) \overset{\text{def'n}}{:=} \lim_{h\to 0} \frac{[f(x+h)]-[f(x)]}{h}$

e.g.: $\frac{d}{dx}[x^5] = 5x^4$

$\frac{d}{dx}[x^4] = 4x^3$

True for $n = 4, 5$.

True for $n = 1/2, -1$.

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THE POWER RULE: (GENERAL VERSION):

$\forall n \in \mathbb{R}$,

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Not quite true for $n = 1$:

$\frac{d}{dx}[x^1] = \frac{d}{dx}[x] = 1 \underset{x\neq 0}{=} 1x^{1-1}$

Proof and study of $x = 0$ case deferred until logarithmic differentiation.

§3.1

Example: Differentiate the function.

$$f(x) = \sqrt{13}$$

SKILL
differentiate constant

$$f'(x) = 0 \blacksquare$$

$$\frac{d}{dx}\left[\sqrt{13}\right] = 0$$

WARNING: $$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$

$$\left[\frac{d}{dx}[\sqrt{x}]\right]_{x:\to 13} = \frac{1}{2\sqrt{13}}$$

$$\nparallel$$

$$\frac{d}{dx}\left[\ [\sqrt{x}]_{x:\to 13}\ \right] = \frac{d}{dx}\left[\sqrt{13}\right] = 0$$

Example: Find an equation of the tangent line to the curve at the given point.

$$y = \sqrt[5]{x}, \quad (32, 2)$$

$$y = x^{1/5}$$

5/5

$$\frac{dy}{dx} = (1/5)x^{(1/5)-1}$$

$$= (1/5)x^{-4/5}$$

SKILL
deriv power fn

$$\left[\frac{dy}{dx}\right]_{x:\to 32} = (1/5)(32)^{-4/5} = \frac{1}{80}$$

SKILL
eq'n tan line

$$y - 2 = \frac{1}{80}(x - 32)$$ ■

Example: Find an equation of the tangent line to the curve at the given point.

$$y = \sqrt[5]{x}, \quad (32, 2)$$

$$y = x^{1/5}$$

$$\frac{dy}{dx} = (1/5)x^{(1/5)-1}$$

$$= (1/5)x^{-4/5}$$

SKILL
deriv power fn

WRONG: $y - 2 = (1/5)x^{-4/5}(x - 32)$

NOT EVEN LINEAR

SKILL
eq'n tan line

$$y - 2 = \frac{1}{80}(x - 32) \blacksquare$$

Example: Find an equation of the tangent line to the curve $y = x \cdot \sqrt[3]{x}$ that is parallel to the line $y = 1 + 4x$.

$\parallel$

$x^{3/3} \cdot x^{1/3} = x^{4/3}$

$$\frac{d}{dx}[x \cdot \sqrt[3]{x}] = \frac{d}{dx}[x^{4/3}] = [4/3][x^{1/3}]$$

SKILL
deriv power fn

$$(3/4) \quad \times \quad \left([4/3][x^{1/3}] = 4 \right)^3$$

$$\left(x^{1/3} = 3 \right)$$

$$x = 3^3 = 27$$

$$[x \cdot \sqrt[3]{x}]_{x:\to 27} = (27)(3) = 81$$

Point: (27, 81) Slope: 4

SKILL
eq'n tan line

Equation: $y - 81 = 4(x - 27)$ ■

SKILL
deriv of powers
Whitman problems
§3.1, p. 48, #1-6

SKILL
words to deriv
Whitman problems
§3.1, p. 48, #7