CALCULUS
Derivatives of exponential functions

Let $f(x) := 10^x$ (with $x = 7 + h$, $x = 7$)

$$f'(7) = \lim_{h \to 0} \frac{10^{7+h} - 10^7}{h}$$

$$= \lim_{h \to 0} \frac{10^7 10^h - 10^7}{h}$$

$$= \lim_{h \to 0} 10^7 \left[\frac{10^h - 1}{h} \right]$$

$$= 10^7 \left[\lim_{h \to 0} \frac{10^h - 1}{h} \right] \quad (??)$$

$$e := \lim_{\substack{x \to \infty \\ h \to 0^+}} \left(1 + \frac{1}{x}\right)^x \quad (h = 1/x)$$

$x :\to 1/h$

$f'(x) = ??$

$f'(7) = ??$

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$$= 10^7 \left[\overset{??}{\boxed{\lim_{h \to 0} \frac{10^h - 1}{h}}} \right]$$

$$e = \lim_{h \to 0^+} (1+h)^{1/h}$$

unneeded

$$e^h \approx (1+h)^{\cancel{1/h}}$$

$$\frac{e^h}{h} \approx \frac{1+h}{h}$$

$$\frac{e^h}{h} - \frac{1+h}{h} \to 0, \quad \text{as } h \to 0$$

$$\| $$

$$\frac{e^h}{h} - \frac{1}{h} - \frac{h}{h}$$

$$\| $$

$$\frac{e^h - 1}{h} - 1$$

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$$\frac{e^h - 1}{h} - 1 \overset{+1}{\quad} \overset{+1}{\quad} \to 0, \quad \text{as } h \to 0$$

$$\frac{e^h - 1}{h} \to 1, \quad \text{as } h \to 0$$

$$\frac{e^h - 1}{h} - 1$$

(logarithmic differentiation)

we'll do 10^x later . . .

$10 :\to e$

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??

similar

$$e = \lim_{h\to 0} (1+h)^{1/h}$$

$$e^h \approx (1+h)^{\cancel{1/h}}$$

$$\frac{e^h}{h} \approx \frac{1+h}{h}$$

$$\frac{e^h - 1}{h} - 1 \to 0, \quad \text{as } h \to 0$$

$$\frac{e^h - 1}{h} \to 1, \quad \text{as } h \to 0$$

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

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equal

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

Let $f(x) := e^x$.

$f'(x) = ??$

$f'(7) = ??$

$7 :\to x$

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Let $f(x) := e^x$.

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$f'(x) = ??$

$f'(7) = ??$

$$\frac{d}{dx}[e^x] = e^x$$

EXAMPLE: Differentiate this expression:

SKILL
deriv exp

$$y = pe^w + \frac{q}{\sqrt{w}} + \frac{r}{w^4}$$

$$\frac{dy}{dw} = p\left[\frac{d}{dw}\left(e^w\right)\right] + q\left[\frac{d}{dw}\left(\frac{1}{\sqrt{w}}\right)\right] + r\left[\frac{d}{dw}\left(\frac{1}{w^4}\right)\right]$$

$$= p\left[\frac{d}{dw}\left(e^w\right)\right] + q\left[\frac{d}{dw}\left(w^{-1/2}\right)\right] + r\left[\frac{d}{dw}\left(w^{-4}\right)\right]$$

$$\frac{d}{dx}[e^x] = e^x$$

$x :\to w$

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$$y = pe^w + \frac{q}{\sqrt{w}} + \frac{r}{w^4}$$

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$$= p\left[\frac{d}{dw}(e^w)\right] + q\left[\frac{d}{dw}\left(w^{-1/2}\right)\right] + r\left[\frac{d}{dw}\left(w^{-4}\right)\right]$$

$$= p\left[e^w\right] + q\left[(-1/2)w^{-3/2}\right] + r\left[-4w^{-5}\right]$$

could stop here, but format of answer different from format of question

$$= pe^w - \frac{q}{2w^{3/2}} - \frac{4r}{w^5}$$

$w^{3/2} = w^{1+(1/2)}$

$= w^1 w^{1/2}$

$= w\sqrt{w}$

$$= pe^w - \frac{q}{2w\sqrt{w}} - \frac{4r}{w^5}$$

$$\frac{d}{dw}[e^w] = e^w$$

STOP