CALCULUS
The quotient rule

Let $u := t^5$ and let $v := e^t$.

Goal: $\frac{d}{dt}[t^5/e^t]$

$\Delta u = (t+\Delta t)^5 - t^5$

$\Delta v = e^{t+\Delta t} - e^t$

Let $w := u/v$, so $w = t^5/e^t$. $\Delta w = \left[(t+\Delta t)^5/e^{t+\Delta t}\right] - \left[t^5/e^t\right]$

$\Delta u = [u]_{t:\to t+\Delta t} - u$

$\Delta v = [v]_{t:\to t+\Delta t} - v$

$u + \Delta u = [u]_{t:\to t+\Delta t}$

$v + \Delta v = [v]_{t:\to t+\Delta t}$

$$\Delta w = [u/v]_{t:\to t+\Delta t} - [u/v]$$

$$= \left[\frac{u+\Delta u}{v+\Delta v}\right] - \left[\frac{u}{v}\right]$$

$$= \left[\frac{u+\Delta u}{v+\Delta v}\right]\left[\frac{v}{v}\right] - \left[\frac{v+\Delta v}{v+\Delta v}\right]\left[\frac{u}{v}\right]$$

1 1

$$= \frac{[u+\Delta u]v - [v+\Delta v]u}{[v+\Delta v]v}$$

COMMON DENOM

$$= \frac{[uv + v(\Delta u)] - [uv + u(\Delta v)]}{[v+\Delta v]v} = \frac{v(\Delta u) - u(\Delta v)}{[v+\Delta v]v}$$

Let $u := t^5$ and let $v := e^t$.

Goal: $\frac{d}{dt}[t^5/e^t]$

$$\triangle u = (t + \triangle t)^5 - t^5 \qquad \triangle v = e^{t+\triangle t} - e^t$$

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$$\triangle u = [u]_{t:\to t+\triangle t} - u \qquad \triangle v = [v]_{t:\to t+\triangle t} - v$$

$$u + \triangle u = [u]_{t:\to t+\triangle t} \qquad v + \triangle v = [v]_{t:\to t+\triangle t}$$

$$\frac{\triangle w}{\triangle w} = \boxed{\frac{v(\triangle u) - u(\triangle v)}{[v + \triangle v]v}}$$

DIVIDE BY $\triangle t$

$$/\!\!/ \ \frac{v(\triangle u) - u(\triangle v)}{[v + \triangle v]v}$$

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$$u + \triangle u = [u]_{t:\to t+\triangle t} \qquad v + \triangle v = [v]_{t:\to t+\triangle t}$$

$$\triangle w = \frac{v(\triangle u) - u(\triangle v)}{[v + \triangle v]v}$$

DIVIDE BY $\triangle t$

$$\frac{\triangle w}{\triangle t} = \frac{v(\triangle u/\triangle t) - u(\triangle v/\triangle t)}{[v + \triangle v]v}$$

LET $\triangle t \to 0$

$$\triangle v = \frac{\triangle v}{\triangle t}\triangle t \to \frac{dv}{dt}0 = 0$$

$$\frac{dw}{dt} = \frac{v(du/dt) - u(dv/dt)}{[v + 0]v}$$

$$= \frac{v(du/dt) - u(dv/dt)}{v^2}$$

Let $u := t^5$ and let $v := e^t$.

$\triangle u = (t + \triangle t)^5 - t^5$

$\triangle v = e^{t+\triangle t} - e^t$

Goal: $\dfrac{d}{dt}[t^5/e^t]$

Let $w := u/v$, so $w = t^5/e^t$. $\triangle w = \left[(t + \triangle t)^5/e^{t+\triangle t}\right] - \left[t^5/e^t\right]$

$$\frac{dw}{dt} = \frac{v(du/dt) - u(dv/dt)}{v^2}$$ ← THE QUOTIENT RULE

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Let $u := t^5$ and let $v := e^t$.

Goal: $\dfrac{d}{dt}[t^5/e^t]$

$\triangle u = (t + \triangle t)^5 - t^5$ $\qquad$ $\triangle v = e^{t+\triangle t} - e^t$

Let $w := u/v$, so $w = t^5/e^t$. $\quad \triangle w = \left[(t + \triangle t)^5/e^{t+\triangle t}\right] - \left[t^5/e^t\right]$

$$\frac{dw}{dt} = \frac{v(du/dt) - u(dv/dt)}{v^2}$$

← THE QUOTIENT RULE

$$= \frac{e^t(5t^4) - t^5(e^t)}{(e^t)^2} = \mathit{etc.}$$

Let $w := u/v$.

$du/dt :\to f'(t)$, $dv/dt :\to g'(t)$

$$\frac{dw}{dt} = \frac{v(du/dt) - u(dv/dt)}{v^2}$$ ← THE QUOTIENT RULE

$w :\to \frac{f(t)}{g(t)}$ $v :\to g(t)$, $u :\to f(t)$

$$\frac{d}{dt}\left[\frac{f(t)}{g(t)}\right] = \frac{[g(t)][f'(t)] - [f(t)][g'(t)]}{[g(t)]^2}$$

$t :\to x$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{[g(x)][f'(x)] - [f(x)][g'(x)]}{[g(x)]^2}$$

functions. . .

$$\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2}$$

§3.4

THE QUOTIENT RULE

THE QUOTIENT RULE

high → low →

$$\left[\frac{f}{g}\right]' = \frac{g\,f' \quad - \quad f\,g'}{g^2}$$

$$\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2}$$

THE QUOTIENT RULE

Low dee high (derivative) less high dee low (derivative),

high → f, low → g

$$\left[\frac{f}{g}\right]' = \frac{g\,f' - f\,g'}{g^2}$$

and underneath,
low squared'll go.

Exercise: Write out the formulas for

$$\frac{d}{ds}\left[\frac{f(s)}{g(s)}\right] \text{ and } \frac{d}{dz}\left[\frac{f(z)}{g(z)}\right]$$

THE QUOTIENT RULE

Low dee high less high dee low,

e.g.:

$$\frac{d}{dt}\left[\frac{e^t}{t^{1/2}}\right] = \frac{[t^{1/2}]\,[e^t] - [e^t]\,[(1/2)t^{-1/2}]}{(t^{1/2})^2}$$

■

and underneath,
low squared'll go.

SKILL
quot rule

Exercise: $\frac{d}{du}\left[\frac{3u^2 - u + 1}{e^u + 5}\right]$

EXAMPLE: Find the derivative of the fn

SKILL
quot rule

$$F(x) = \frac{x - 3x\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$$

in two ways: (a) by using the Quotient Rule and (b) by simplifying first.

$$\frac{\text{l.c. of powers of } x}{\text{power of } x} = \text{l.c. of powers of } x$$

(a) $$F(x) = \frac{x - 3x^{5/3}}{x^{2/3}}$$

$$F'(x) = \frac{[x^{2/3}][1 - 5x^{2/3}] - [x - 3x^{5/3}][(2/3)x^{-1/3}]}{x^{4/3}}$$

(b) $$F(x) = \left[\frac{x}{x^{2/3}}\right] - 3\left[\frac{x^{5/3}}{x^{2/3}}\right] = x^{1/3} - 3x$$

$$F'(x) = (1/3)x^{-2/3} - 3$$

EXAMPLE: Find the derivative of the fn

SKILL
quot rule

$$F(x) = \frac{x - 3x\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$$

in two ways: (a) by using the Quotient Rule and (b) by simplifying first.

(c) Show that your answers are equivalent.

(a) $$F'(x) = \frac{[x^{2/3}][1 - 5x^{2/3}] - [x - 3x^{5/3}][(2/3)x^{-1/3}]}{x^{4/3}}$$

$$F'(x) = \frac{[x^{2/3}][1 - 5x^{2/3}] - [x - 3x^{5/3}][(2/3)x^{-1/3}]}{x^{4/3}}$$

(b)

$$(F'(x) = (1/3)x^{-2/3} - 3 - 3$$

EXAMPLE: Find the derivative of the fn

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$$F(x) = \frac{x - 3x\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$$

in two ways: (a) by using the Quotient Rule and (b) by simplifying first.

(c) Show that your answers are equivalent. ☺

(d) Which method do you prefer? (b) ■

(a) $$F'(x) = \frac{[x^{2/3}][1 - 5x^{2/3}] - [x - 3x^{5/3}][(2/3)x^{-1/3}]}{x^{4/3}}$$

$$= \frac{[x^{2/3} - 5x^{4/3}] - [(2/3)x^{2/3} - 2x^{4/3}]}{x^{4/3}}$$

$$= [x^{-2/3} - 5] - [(2/3)x^{-2/3} - 2]$$

$$= x^{-2/3} - (2/3)x^{-2/3} - 5 + 2$$

$$\parallel$$

(b) $$F'(x) = (1/3)x^{-2/3} - 3$$

EXAMPLE: Differentiate: $y = \frac{e^{2x}}{1-x}$

SKILL
quot rule

$$\frac{dy}{dx} = \frac{[1-x][2e^{2x}] - [e^{2x}][-1]}{(1-x)^2}$$

$$\frac{d}{dx}[e^{2x}] = \frac{d}{dx}[e^x e^x]$$

$$= [e^x][e^x] + [e^x][e^x]$$

$$= e^{2x} + e^{2x}$$

$$= 2e^{2x}$$

EXAMPLE: Differentiate: $y = \dfrac{4t}{(t+3)^2}$

SKILL quot rule

$$y = \frac{4t}{t^2 + 6t + 9}$$

$$\frac{dy}{dx} = \frac{[t^2 + 6t + 9][4] - [4t][2t + 6]}{(t^2 + 6t + 9)^2}$$

EXAMPLE: Differentiate: $g(t) = \frac{4t - \sqrt{t}}{t^{2/3}}$ SKILL quot rule

$$g'(t) = (4/3)t^{-2/3} + (1/6)t^{-7/6}$$ ■

Exercise: Check that these are equal.

$$g(t) = \frac{4t - t^{1/2}}{t^{2/3}} = 4t^{1/3} - t^{-1/6}$$

$$g'(t) = \frac{[t^{2/3}][4 - (1/2)t^{-1/2}] - [4t - t^{1/2}][(2/3)t^{-1/3}]}{t^{4/3}}$$

alternate solution. . .

$$(1/2) - (2/3) = (3/6) - (4/6) = -1/6$$

EXAMPLE: Differentiate: $f(x) = \dfrac{2 + xe^x}{x - e^x}$ SKILL quot rule

$$f(x) = \frac{[x - e^x][0 + x(e^x) + (1)e^x] - [2 + xe^x][1 - e^x]}{(x - e^x)^2}$$

SKILL
quot rule

Whitman problems
§3.4, p. 56, #1-4

SKILL
eq'n tan line

Whitman problems
§3.4, p. 56, #5-6,8

SKILL
deriv gen rat'l

Whitman problems
§3.4, p. 56, #7

SKILL
1-jets to 1-jet quot

Whitman problems
§3.4, p. 56, #9

STOP