CALCULUS
Chain rule problems

EXAMPLE: Differentiate

(a) $y = \cos(x^3)$ (b) $y = \cos^3 x$

(a) $\frac{dy}{dx} = [-\sin(\ x^3\ )][3x^2] = -3x^2 \sin(x^3)$

(b) $\frac{dy}{dx} = [3(\ \cos x\ )^2][-\sin x]$

$= -3(\sin x)(\cos^2 x)$

YOU CAN SIMPLIFY A LITTLE . . .

EXAMPLE: Differentiate $y = (x^6 - 1)^{75}$.

EXPAND? I DON'T THINK SO!

$$\frac{dy}{dx} = [\,75(\ x^6 - 1\,)^{74}\,][6x^5]$$
$$= 450x^5(x^6 - 1)^{74}$$

YOU CAN SIMPLIFY A LITTLE . . .

EXAMPLE: Find the derivative of the function.

$$F(x) = (5x - x^3)^{45}$$

EXPAND? I DON'T THINK SO!

$$F'(x) = [\,45(5x - x^3)^{44}\,][5 - 3x^2] \quad \blacksquare$$

EXAMPLE: Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.] Then find the derivative dy/dx.

$$y = \cot(6x)$$

$u = g(x) = 6x$ $\qquad$ $f = \cot$, so $y = f(u) = \cot u$,

so $y = \cot(6x)$

$$\frac{dy}{dx} \underset{\text{Chain Rule}}{=} \left[\frac{dy}{du}\right]\left[\frac{du}{dx}\right]$$

$$= \left[-\csc^2 u\right][6]$$

$$= \left[-\csc^2(6x)\right][6] \blacksquare$$

MY WAY: $\dfrac{d}{dx}[\cot(6x)]$

$$= \left[-\csc^2(6x)\right][6]$$

EXAMPLE: Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.] Then find the derivative dy/dx.

$$y = \csc(\cos x)$$

$$u = \cos x, \qquad f = \csc$$

$$y = f(u) = \csc u = \csc(\cos x)$$

$$\frac{dy}{dx} \underset{\text{Chain Rule}}{=} \left[\frac{dy}{du}\right]\left[\frac{du}{dx}\right]$$

$$= [-(\csc u)(\cot u)]\,[-\sin x]$$

$$= [-(\csc(\cos x))(\cot(\cos x))]\,[-\sin x] \blacksquare$$

MY WAY:

$$\frac{d}{dx}[\csc(\cos x)] = [-(\csc(\cos x))(\cot(\cos x))]\ [-\sin x]$$

EXAMPLE: Find $F'(x)$ if

$$F(x) = \sqrt{x^3 + 4}.$$

$$F(x) = (x^3 + 4)^{1/2}$$

$$F'(x) = (1/2)(x^3 + 4)^{-1/2}(3x^2)$$

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EXAMPLE: Find the derivative of the fn.

$$f(t) = \sqrt[7]{3 + \csc t}$$

$$f'(t) = [\,\tfrac{1}{7}(\,3 + \csc t\,)^{-6/7}][-(\csc t)(\cot t)]$$

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EXAMPLE: Find $f'(x)$ if $f(x) = \dfrac{1}{\sqrt[3]{2x^2 - x - 4}}$.

$$f(x) = (2x^2 - x - 4)^{-1/3}$$

$$f'(x) = (-1/3)(2x^2 - x - 4)^{-4/3}(4x - 1) \blacksquare$$

EXAMPLE: Find the derivative of the fn.

$$f(t) = \sqrt{\frac{9t-2}{t^3-7}}$$

$$f(t) = \left[\frac{9t-2}{t^3-7}\right]^{1/2}$$

$$f'(t) = \frac{1}{2}\left[\frac{9t-2}{t^3-7}\right]^{-1/2}\left[\frac{(t^3-7)(9)-(9t-2)(3t^2)}{(t^3-7)^2}\right]$$

EXAMPLE:

If $f(x) = \sin(\tan(\cos x))$, then $f'(x) = \cdots$

$$f'(x) = [\cos(\tan(\cos x))]\,[\sec^2(\cos x)]\,[-\sin x]$$

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EXAMPLE: Find the derivative of the fn.

$$f(x) = \cos(\sin(\cot x))$$

$$f'(x) = [-\sin(\sin(\cot x))][\cos(\cot x))][-\csc^2 x] \blacksquare$$

EXAMPLE: Find the equation of the tangent line to the curve at the given point.

$$y = (\cos x) + (\cos^2 x)\ , \qquad \left(\frac{\pi}{4}\ ,\ \frac{1+\sqrt{2}}{2}\right)$$

$$\frac{dy}{dx} = (-\sin x) + 2(\cos x)(-\sin x)$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right)$$

$$\left[\frac{dy}{dx}\right]_{x:\to\pi/4} = -\frac{\sqrt{2}}{2} + 2\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = -\frac{2+\sqrt{2}}{2}$$

$$y - \frac{1+\sqrt{2}}{2} = -\frac{2+\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) \ \blacksquare$$

EXAMPLE: Find the x-coordinates of the points on the curve $y = (\cos(2x)) - 2$ at which the tangent line is horizontal.

$$\frac{dy}{dx} = -[\sin(2x)]\,[2] - 0 = -2\sin(2x)$$

$$-2\sin(2x) = 0$$

$$\sin(2x) = 0$$

$$2x = n\pi, \qquad n \in \mathbb{Z}$$

$$x = \frac{n\pi}{2}, \qquad n \in \mathbb{Z}$$

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EXAMPLE: If $G(x) = f(x \cdot f(x \cdot f(x)))$, where $f(2) = 6$, $f'(2) = 3$, $f(12) = 6$, $f'(12) = 7$, find $G'(2)$.

$$G'(x) = [f'(x \cdot f(x \cdot f(x)))][f(x \cdot f(x)) + x \cdot [f'(x \cdot f(x))][f(x) + x \cdot f'(x)]]$$

EXAMPLE: If $G(x) = f(x \cdot f(x \cdot f(x)))$, where $f(2) = 6$, $f'(2) = 3$, $f(12) = 6$, $f'(12) = 7$, find $G'(2)$.

$$G'(\overset{2}{x}) = [f'(\overset{2}{x} \cdot f(\overset{2}{x} \cdot f(\overset{2}{x})))][f(\overset{2}{x} \cdot f(\overset{2}{x})) +$$

$$\overset{2}{x} \cdot [f'(\overset{2}{x} \cdot f(\overset{2}{x}))][f(\overset{2}{x}) +$$

$$\overset{2}{x} \cdot f'(\overset{2}{x})$$

$$]$$

$$]$$

EXAMPLE: If $G(x) = f(x \cdot f(x \cdot f(x)))$, where $f(2) = 6$, $f'(2) = 3$, $f(12) = 6$, $f'(12) = 7$, find $G'(2)$.

$$G'(x) = [f'(x \cdot f(x \cdot f(x)))]\,[f(x \cdot f(x)) + x \cdot [f'(x \cdot f(x))]\,[f(x) + x \cdot f'(x)]]$$

$$G'(2) = [7][6 + 2 \cdot [7][6 + 2 \cdot 3]]$$

EXAMPLE: If $G(x) = f(x \cdot f(x \cdot f(x)))$, where $f(2) = \boxed{6}$, $f'(2) = \boxed{3}$, $f(12) = \boxed{6}$, $f'(12) = \boxed{7}$, find $G'(2)$.

$$G'(x) = [f'(x \cdot f(x \cdot f(x)))][f(x \cdot f(x)) + x \cdot [f'(x \cdot f(x))][f(x) + x \cdot f'(x)]]$$

$$G'(2) = [7][6 + 2 \cdot [7][6 + 2 \cdot 3]]$$

$$= [7][6 + 2 \cdot [7][12]]$$

§3.5

EXAMPLE: If $G(x) = f(x \cdot f(x \cdot f(x)))$, where $f(2) = 6$, $f'(2) = 3$, $f(12) = 6$, $f'(12) = 7$, find $G'(2)$.

$$G'(2) = [7][6 + 2 \cdot [7][12]]$$
$$= [7][6 + 2 \cdot [84]]$$

$$G'(2)$$

$$= [7][6 + 2 \cdot [7][12]]$$

EXAMPLE: If $G(x) = f(x \cdot f(x \cdot f(x)))$, where $f(2) = \boxed{6}$, $f'(2) = 3$, $f(12) = \boxed{6}$, $f'(12) = 7$, find $G'(2)$.

$$
\begin{aligned}
G'(2) &= [7][6 + 2 \cdot [7][12]] \\
&= [7][6 + 2 \cdot [84]] \\
&= [7][6 + 168] \\
&= [7][174] \\
&= 1218 \blacksquare
\end{aligned}
$$

SKILL
Misc differentiation
Whitman problems
§3.5, p. 60–61, #1-35

SKILL
eq'n tan line
Whitman problems
§3.5, p. 60–61, #36-40

STOP