CALCULUS
Indefinite integration

cf. §7.2, p. 149 The *set* of *all* antiderivatives of $f(x)$ w.r.t. x is denoted $\boxed{\int f(x)\,dx}$.

INDEFINITE INTEGRALS, PROPERTIES & EXAMPLES:

$$\int c[f(x)]\,dx = c\int f(x)\,dx$$

e.g.: $\int 3x^2\,dx = 3\int x^2\,dx$

$$\{x^3, x^3+8, x^3-4, \ldots\} \qquad 3\{\tfrac{1}{3}x^3+2, \tfrac{1}{3}x^3-1, \tfrac{1}{3}x^3+100, \ldots\}$$

$$\int 3x^2\,dx = x^3 + C$$

$$3\int x^2\,dx = 3(\tfrac{1}{3}x^3 + C)$$

$$\{3(\tfrac{1}{3}x^3+C)\} = \{x^3+3C\} = \{x^3+C\}$$

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Looks a little strange. . .
but remember that C varies.

$$\{3(\tfrac{1}{3}x^3 + C)\} = \boxed{\{x^3 + 3C\} = \{x^3 + C\}}$$

§8.1

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INDEFINITE INTEGRALS, PROPERTIES & EXAMPLES:

$$\left.\begin{array}{l} \displaystyle\int c[f(x)]\,dx = c\int f(x)\,dx \\ \boxed{\displaystyle\int ([f(x)]+[g(x)])\,dx} = \boxed{\left[\displaystyle\int f(x)\,dx\right]} + \boxed{\left[\displaystyle\int g(x)\,dx\right]} \end{array}\right\} \int \text{ is linear}$$

If you add
each of these expressions
to each of these.
then the resulting set of expressions
will be this set.

WARNING: $\int$ is NOT multiplicative, *i.e.*, $\int fg \neq \left[\int f\right]\left[\int g\right]$.

There's a weak product rule, called "integration by parts", covered in MATH 1272.

INDEFINITE INTEGRALS, PROPERTIES & EXAMPLES:

$$\left.\begin{aligned}\int c[f(x)]\,dx &= c\int f(x)\,dx\\ \int([f(x)]+[g(x)])\,dx &= \left[\int f(x)\,dx\right]+\left[\int g(x)\,dx\right]\end{aligned}\right\}\int \text{ is linear}$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx \overset{x\neq 0}{=} \frac{x^{n+1}}{n+1} + C, \text{ provided } n \neq -1$$

$$\int \frac{1}{x}\,dx \overset{y}{=} [\ln(|x|)] + C \qquad \int \frac{1}{x}\,dx = \begin{cases}(\ln(x)) + A, & \text{if } x > 0\\ (\ln(-x)) + B, & \text{if } x < 0\end{cases}$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

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$$\boxed{\int \frac{1}{x}\,dx = [\ln(|x|)] + C}$$

sloppy

$$\int \frac{1}{x^2+1}\,dx = (\arctan x) + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = (\arcsin x) + C$$

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$$\int \cos x\,dx = (\sin x) + C \qquad \int \sin x\,dx = -(\cos x) + C$$

$$\int \sec^2 x\,dx \overset{\text{sloppy}}{=} (\tan x) + C \qquad \int \csc^2 x\,dx \overset{\text{sloppy}}{=} -(\cot x) + C$$

$$\int (\sec x)(\tan x)\,dx \overset{\text{sloppy}}{=} (\sec x) + C$$

$$\int (\csc x)(\cot x)\,dx \overset{\text{sloppy}}{=} -(\csc x) + C$$

EXAMPLE: Verify by differentiation that

$$\int x(\sin x)\,dx = -x\cos x + \sin x + C$$

is correct.

$$\frac{d}{dx}[-x\cos x + \sin x + C]$$

$$= \cancel{(-1)(\cos x)} + (+x)(+\sin x) + \cancel{\cos x} + 0$$

$$= x(\sin x) \blacksquare$$

SKILL
verify indef int

EXAMPLE: Verify by differentiation that

$$\int \frac{kx}{\sqrt{ax+b}}\,dx = \frac{2k}{3a^2}(ax-2b)\sqrt{ax+b}+C$$

is correct.

$$\frac{d}{dx}\left[\boxed{\frac{2k}{3a^2}}(ax-2b)\sqrt{ax+b}\,\cancel{+C}\right] = \frac{2k}{3a^2}\left[\frac{d}{dx}\left((ax-2b)\sqrt{ax+b}\right)\right]$$

$$= \left[\frac{2k}{3a^2}\right]\left[(ax-2b)\left(\frac{a}{2\boxed{\sqrt{ax+b}}}\right)+(a)\left(\boxed{\sqrt{ax+b}}\right)\right]$$

$2 \cdot \boxed{\frac{a}{2}}$

$\boxed{a}$

$\frac{ax+b}{\boxed{\sqrt{ax+b}}}$

$$= \left[\frac{2k}{3a^2\boxed{\sqrt{ax+b}}}\right]\left[(ax-2b)\boxed{\frac{a}{2}}+\boxed{a}(ax+b)\right]$$

$$= \left[\frac{\cancel{2}k}{3a^{\cancel{2}}\sqrt{ax+b}}\right]\left[\boxed{\frac{\cancel{a}}{\cancel{2}}}\right]\left[(ax-2b)\underline{+2(ax+b)}\right]$$

$$= \left[\frac{k}{3a\sqrt{ax+b}}\right]\underbrace{\left[ax-\cancel{2b}+2ax+\cancel{2b}\right]}_{3ax} = \frac{k(\cancel{3a}x)}{\cancel{3a}\sqrt{ax+b}} \quad ■$$

SKILL
verify indef int

EXAMPLE: Compute $\int v^2(v^2-1)^3\,dv$.

$$\int v^2(v^2-1)^3\,dv$$

$$\|$$

$$\int v^2(v^6-3v^4+3v^2-1)\,dv$$

$$\|$$

$$\int v^8-3v^6+3v^4-v^2\,dv$$

$$\|$$

$$\left[\frac{v^9}{9}\right]-3\left[\frac{v^7}{7}\right]+3\left[\frac{v^5}{5}\right]-\left[\frac{v^3}{3}\right]+C \;\blacksquare$$

SKILL
find indef int

EXAMPLE: Compute $\int [\csc t][(\csc t) + (\cot t)]\, dt$.

$$\int [\csc t][(\csc t) + (\cot t)]\, dt = \int \csc^2 t + (\csc t)(\cot t)\, dt \overset{\text{sloppy}}{=} -\cot t - \csc t + C$$

SKILL
find indef int

EXAMPLE: Compute $\displaystyle\int \frac{\sin(2x)}{\cos x}\,dx$.

$$\| $$

$$\int \frac{2(\sin x)\cancel{(\cos x)}}{\cancel{\cos x}}\,dx$$

provided $\cos x \neq 0$,
i.e., provided

$$\| \quad x \neq \ldots, -2\pi + \tfrac{\pi}{2}, -\pi + \tfrac{\pi}{2}, \tfrac{\pi}{2}, \pi + \tfrac{\pi}{2}, 2\pi + \tfrac{\pi}{2}, \ldots$$

$$2\int \sin x\,dx$$

$$\|$$

$$-2\cos x + C$$

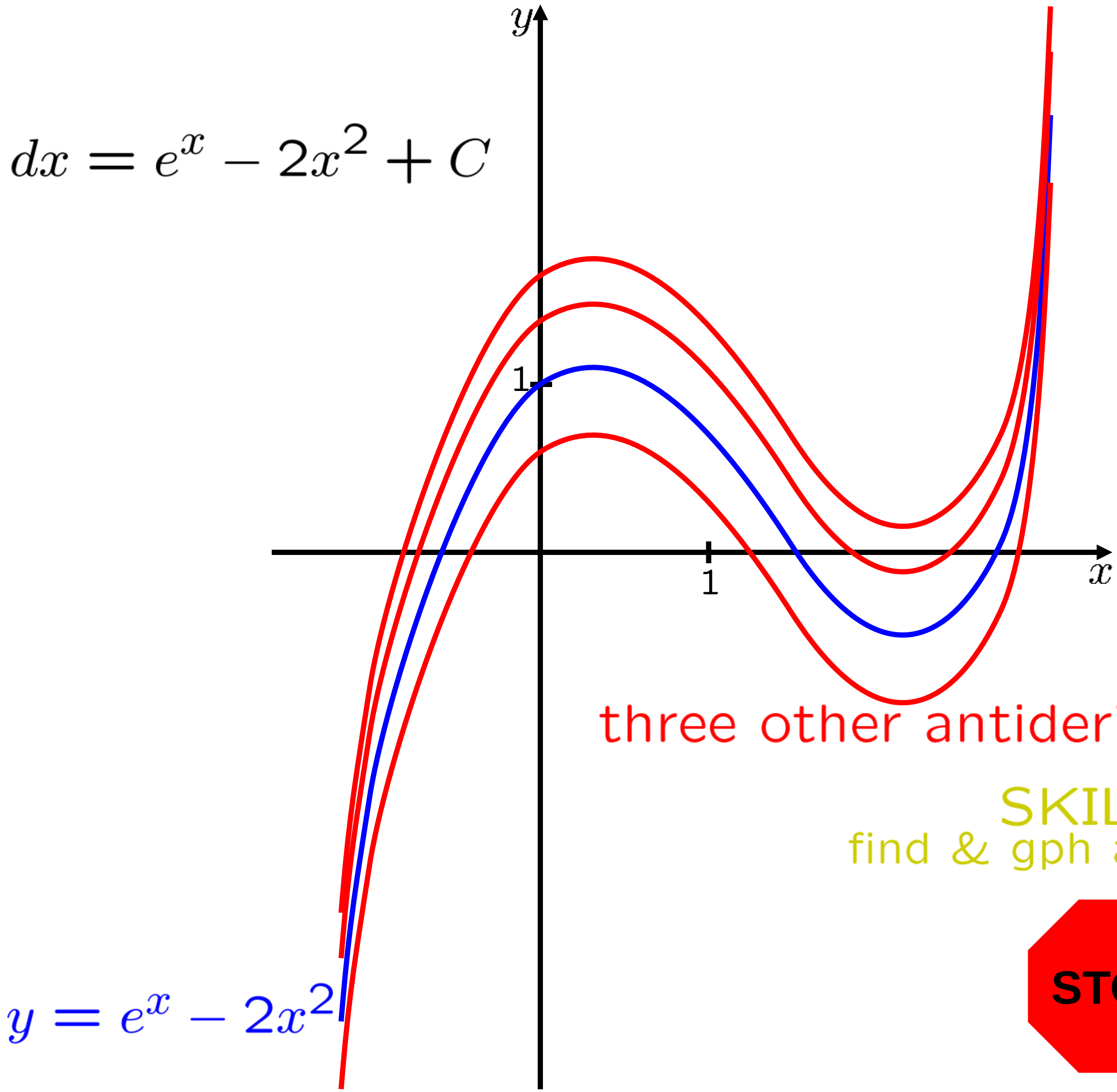
■

SKILL
find indef int

EXAMPLE: Compute $\int (e^x - 4x)\,dx$.

Graph several of the antiderivatives.

$$\int (e^x - 4x)\,dx = e^x - 2x^2 + C$$



SKILL
find & gph antiderivs

STOP