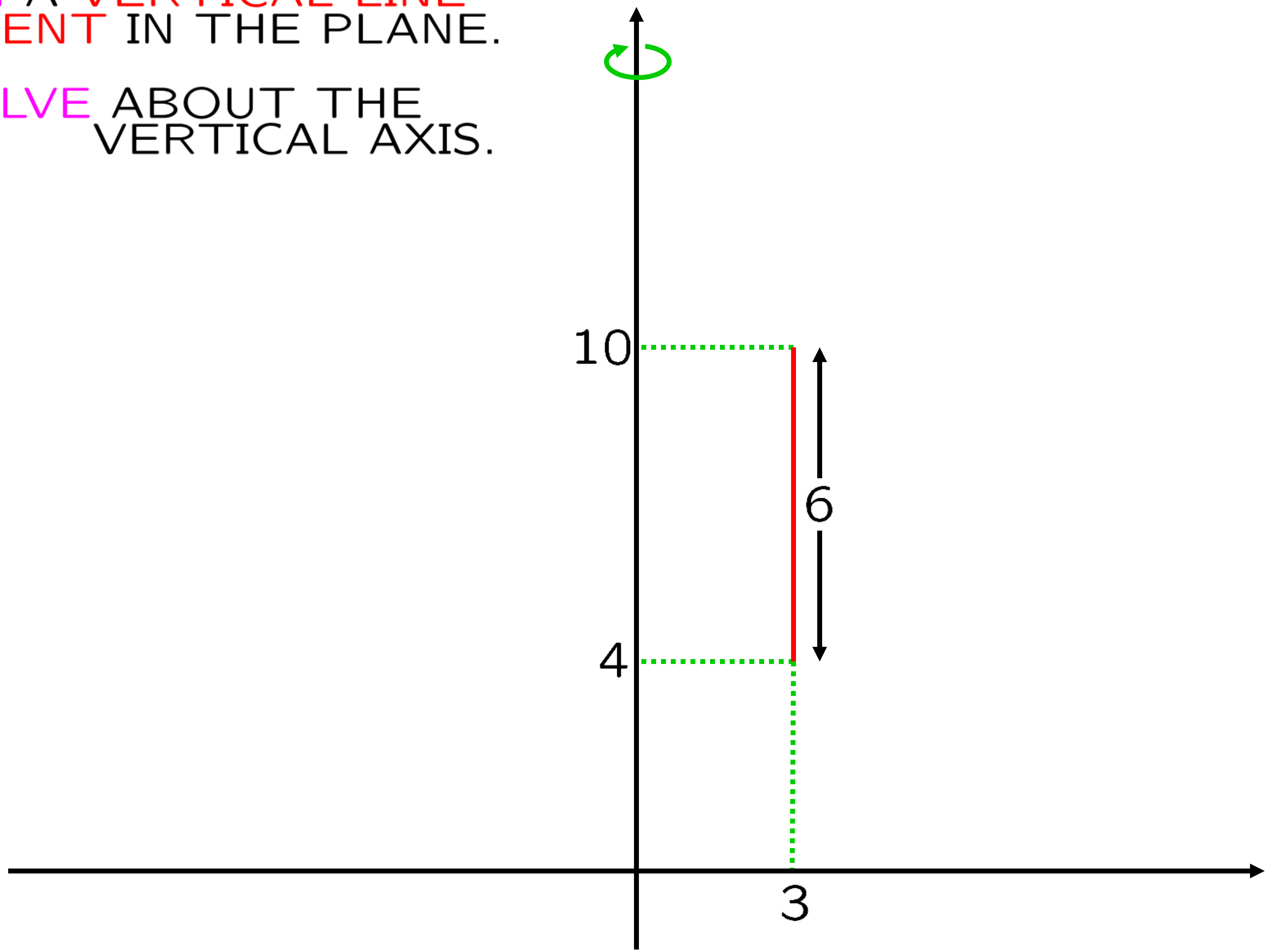
CALCULUS

Volume by cylindrical shells

FORM A VERTICAL LINE SEGMENT IN THE PLANE.

REVOLVE ABOUT THE VERTICAL AXIS.



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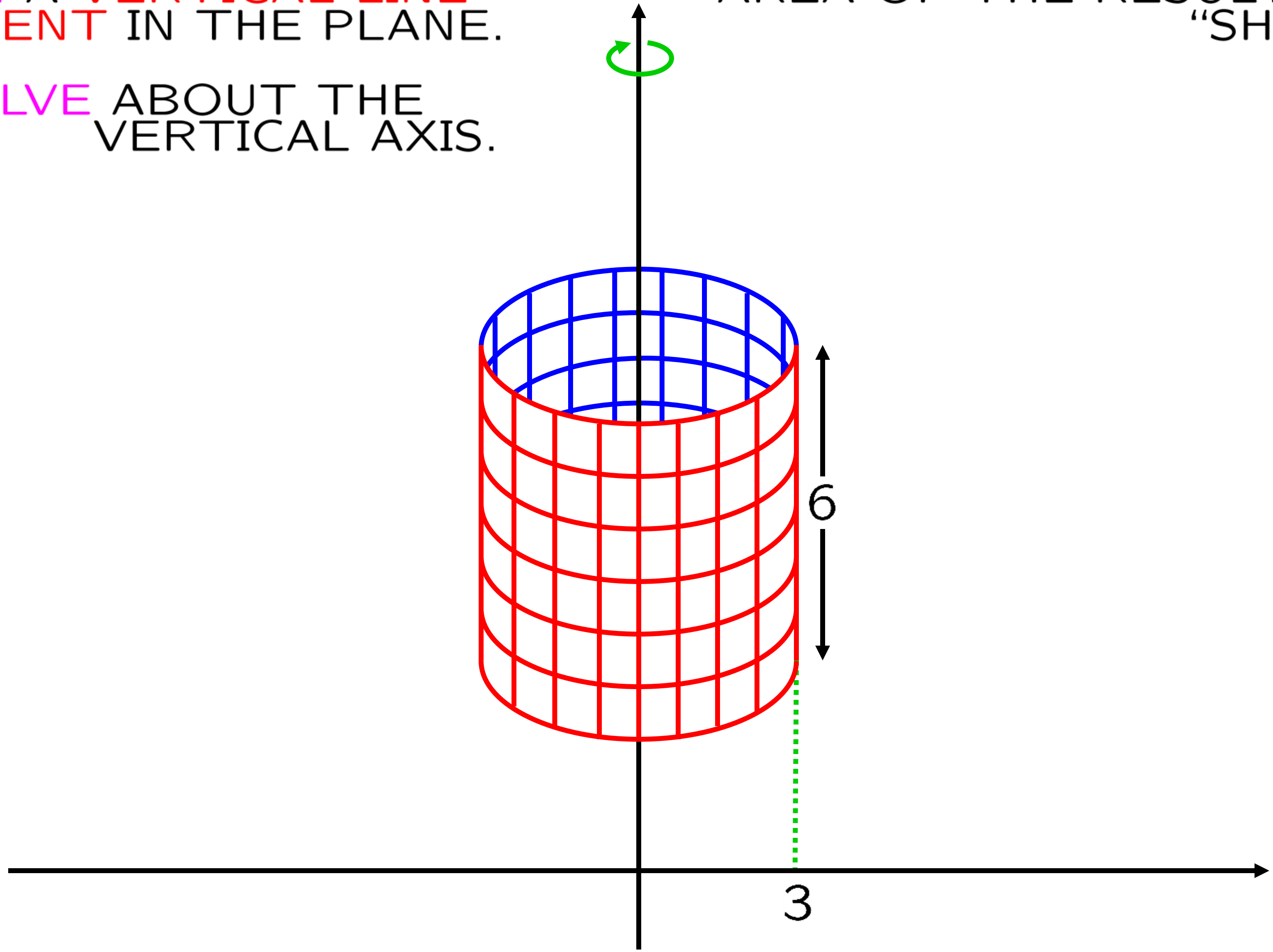
AREA OF THE RESULTING "SHELL"?



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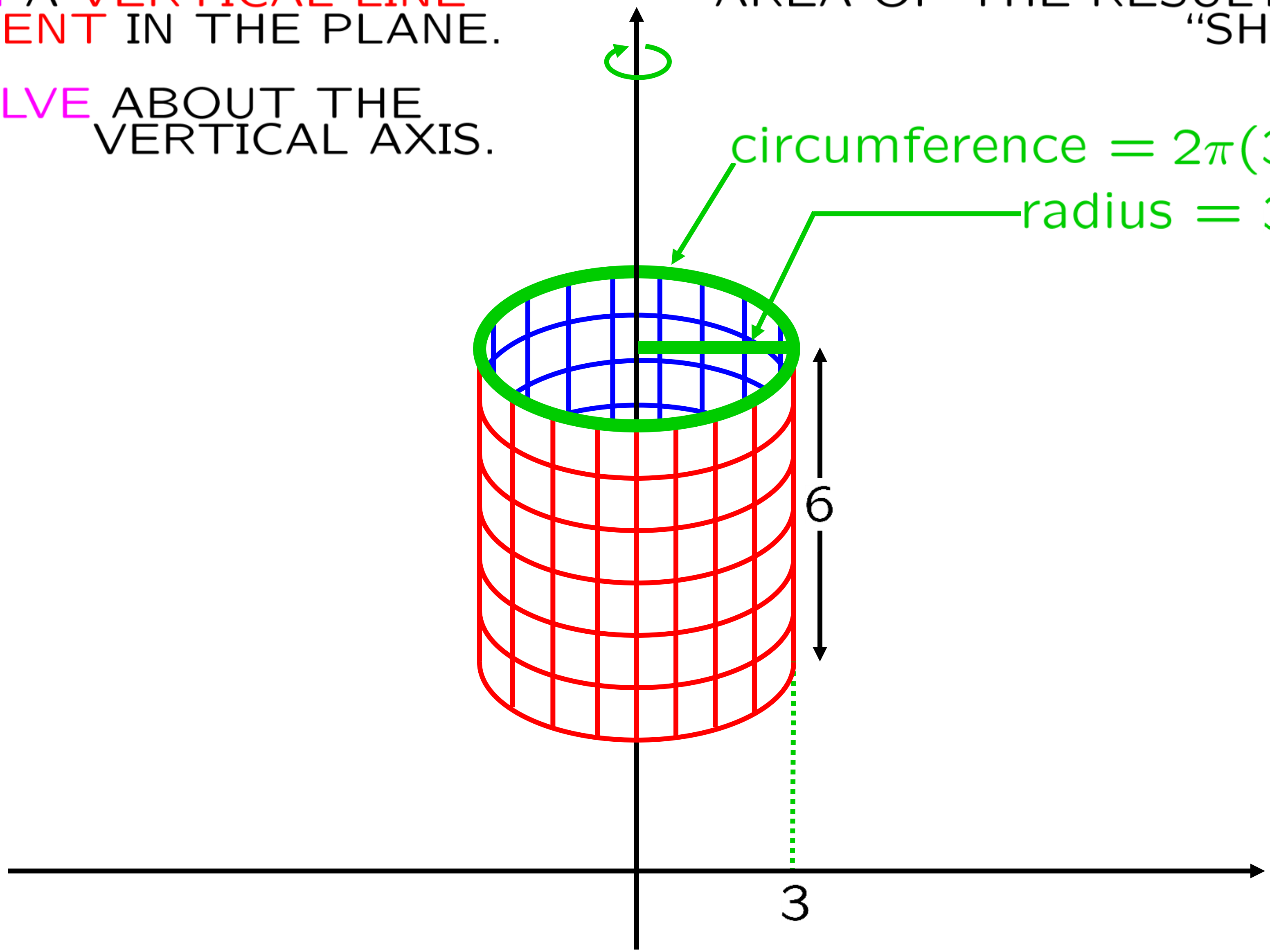

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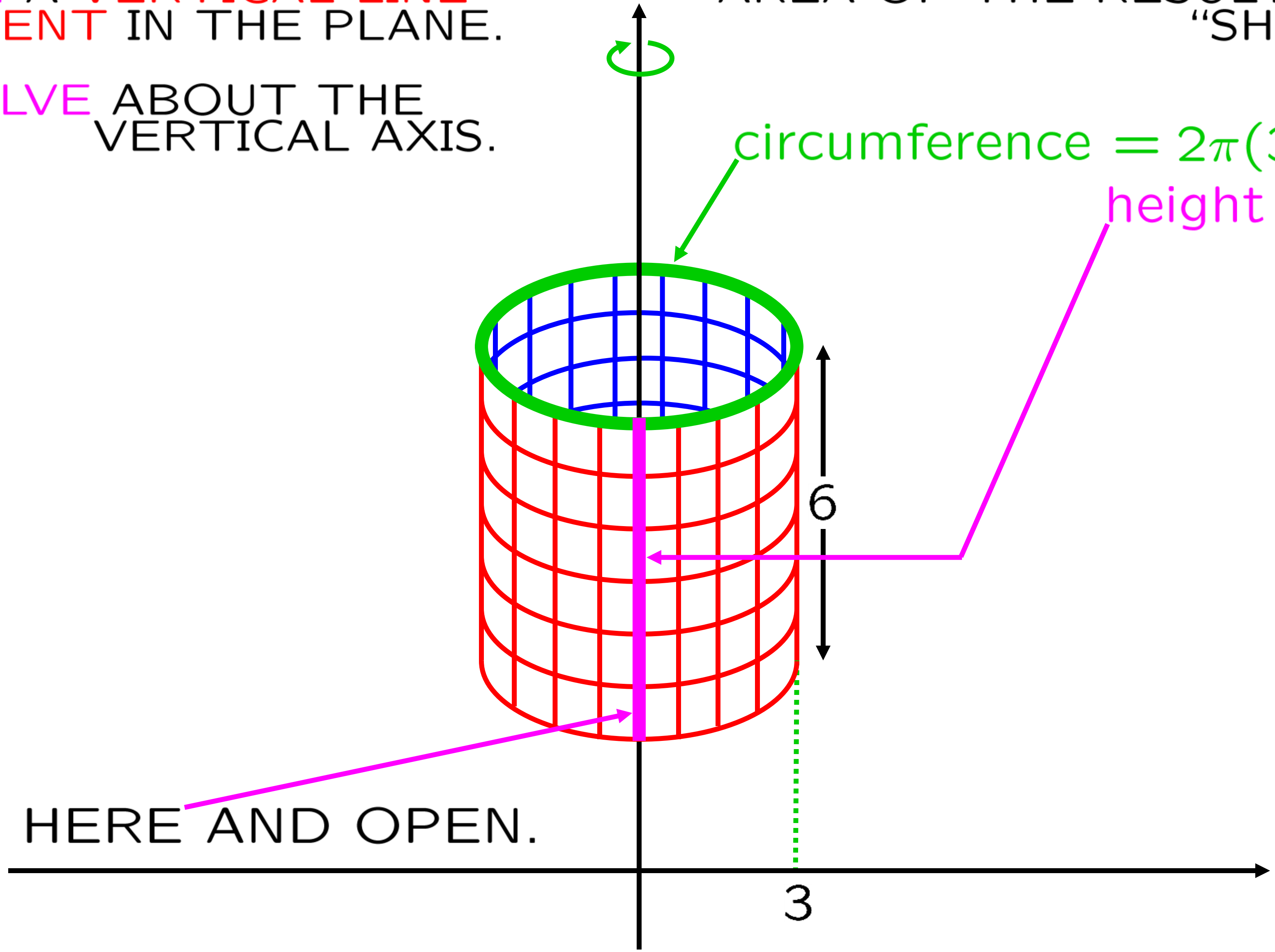

FORM A VERTICAL LINE SEGMENT IN THE PLANE.

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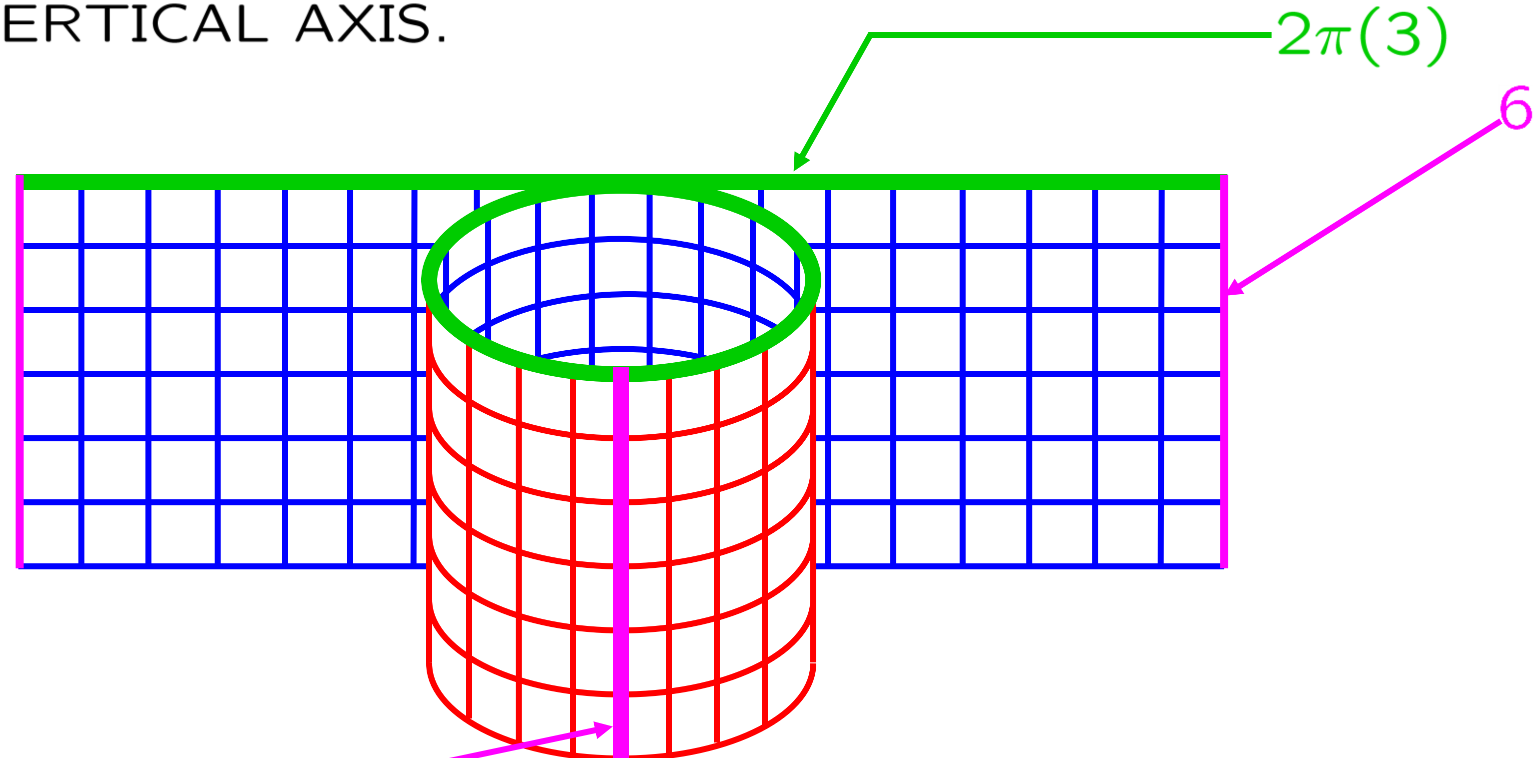
AREA OF THE RESULTING "SHELL"?

ANSWER: $[2\pi(3)][6]$



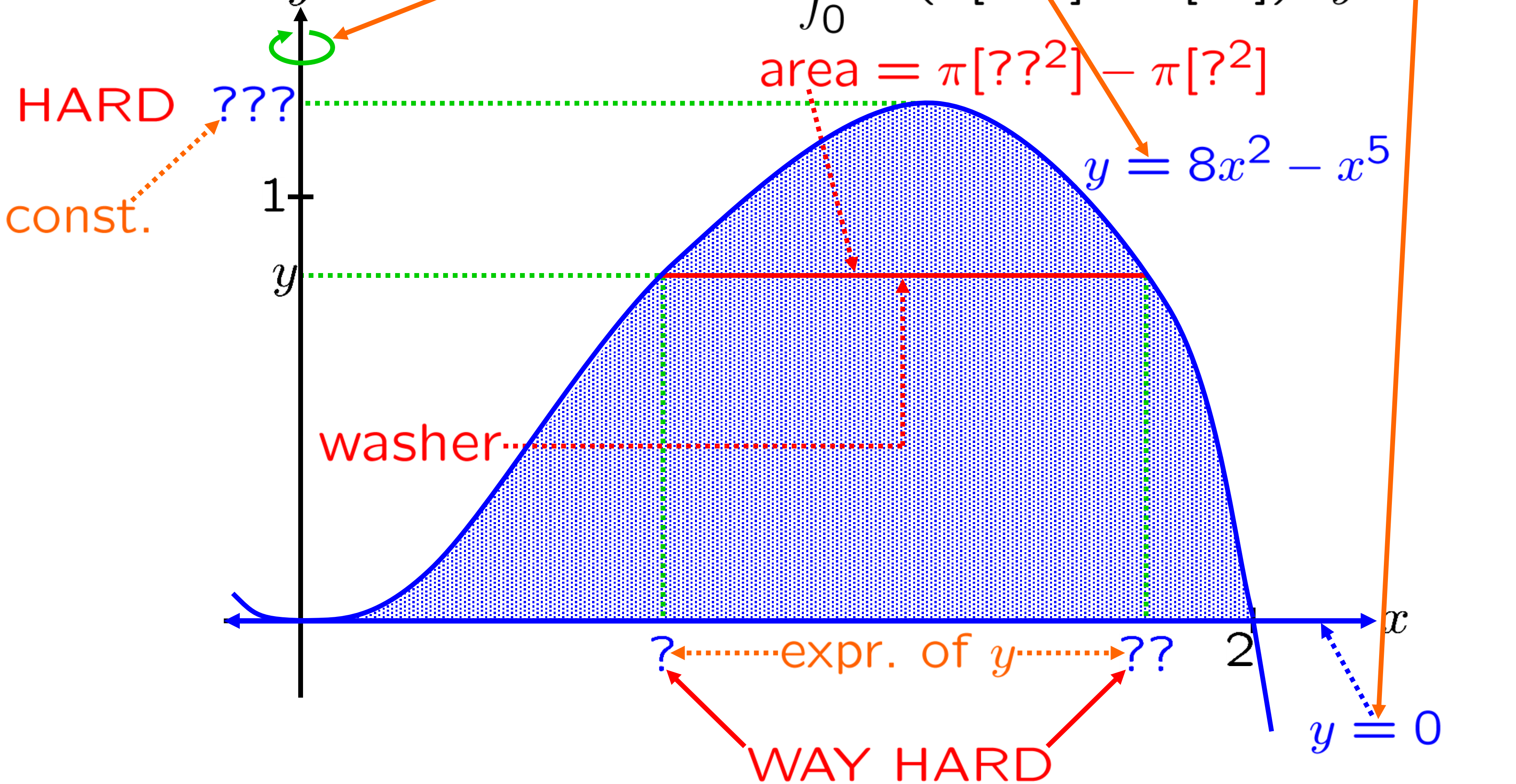
CUT HERE AND OPEN.

AREA OF SHELL = [circumference][height]

EXAMPLE: Find the volume of the solid formed by revolving, about the y-axis, the region bounded by $y = 8x^2 - x^5$ and $y = 0$.

Easier solution. . .

$$\text{volume} = \int_0^{???} (\pi[??^2] - \pi[?^2])\, dy$$



EXAMPLE: Find the volume of the solid formed by revolving, about the y-axis, the region bounded by $y = 8x^2 - x^5$ and $y = 0$.

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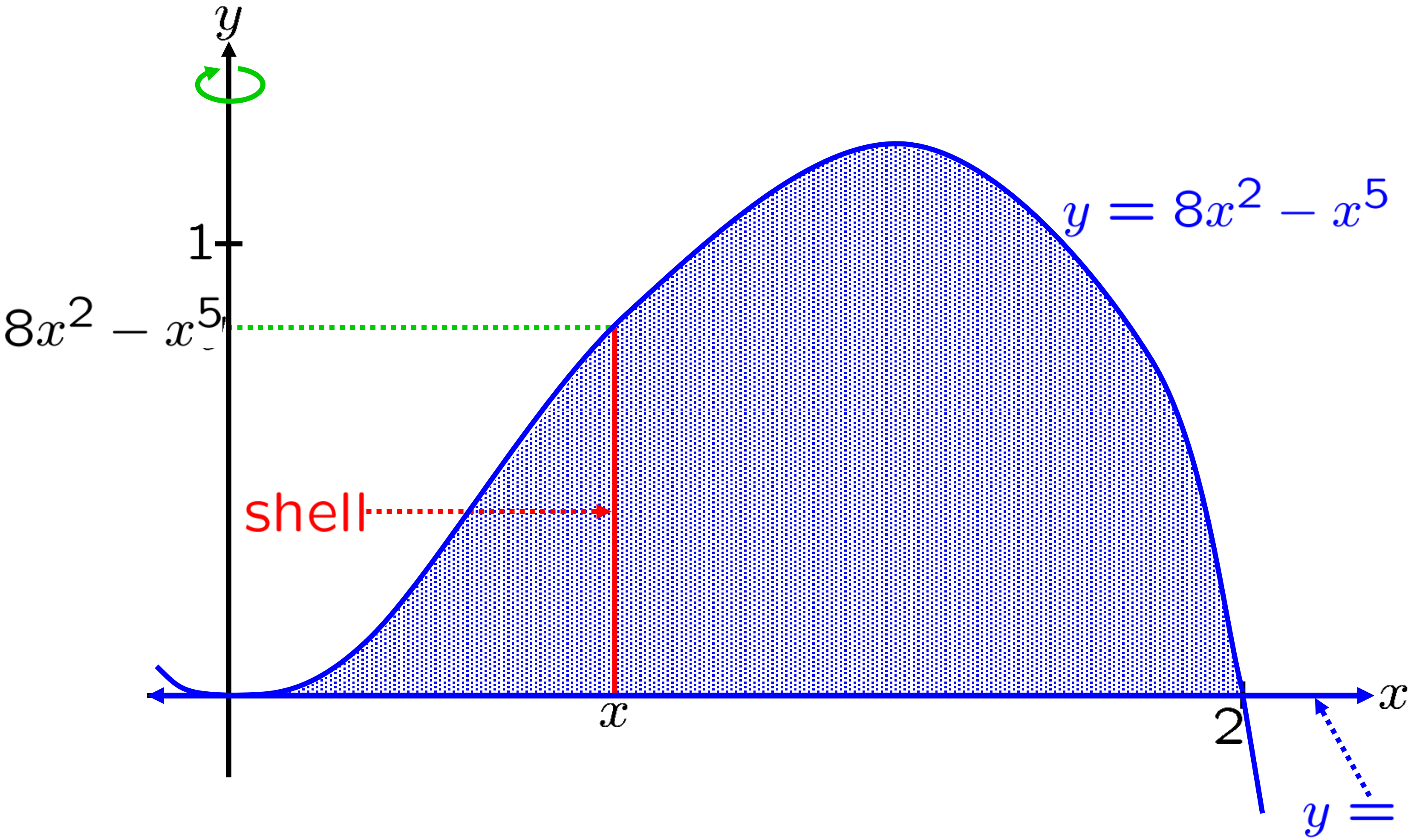

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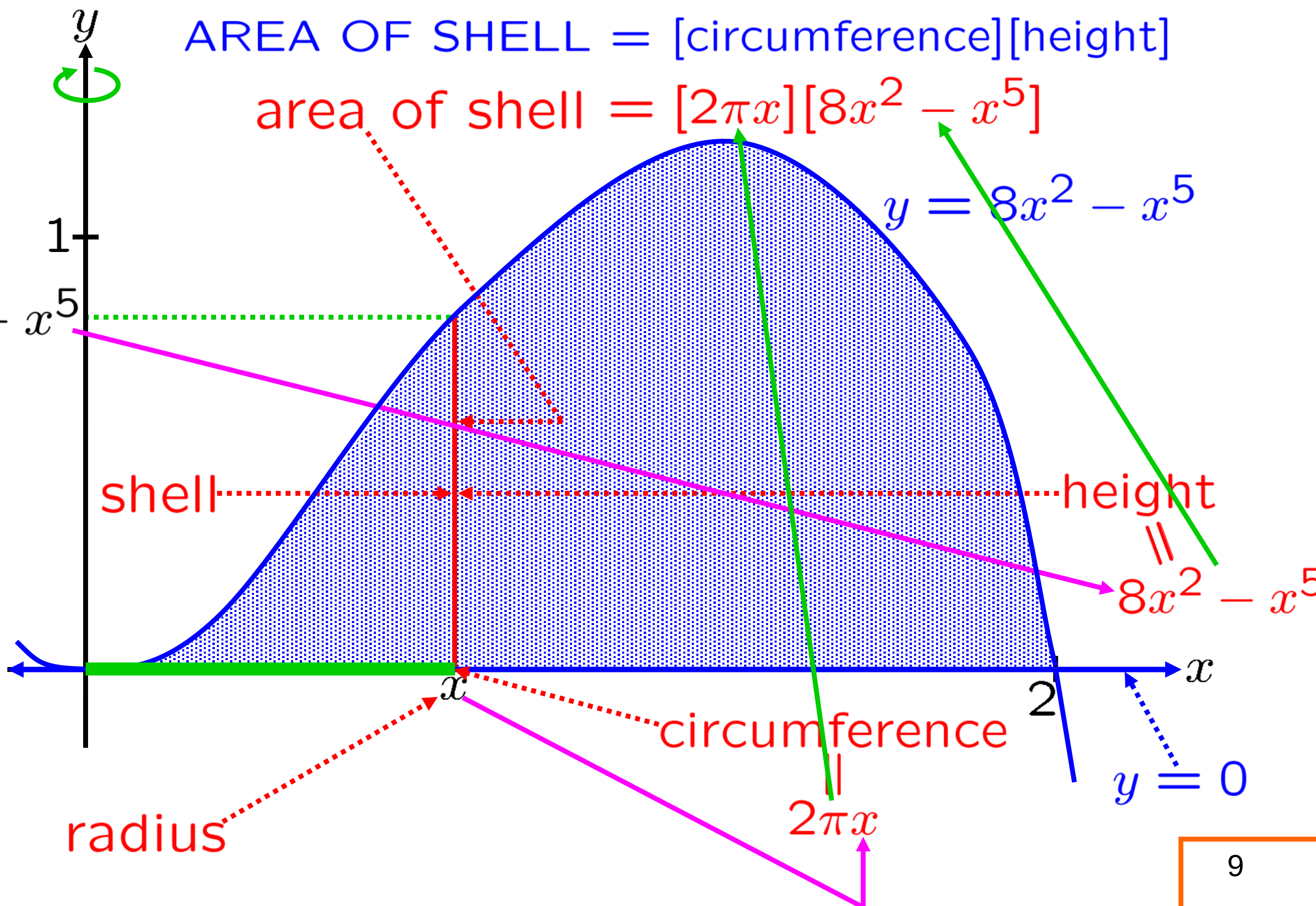
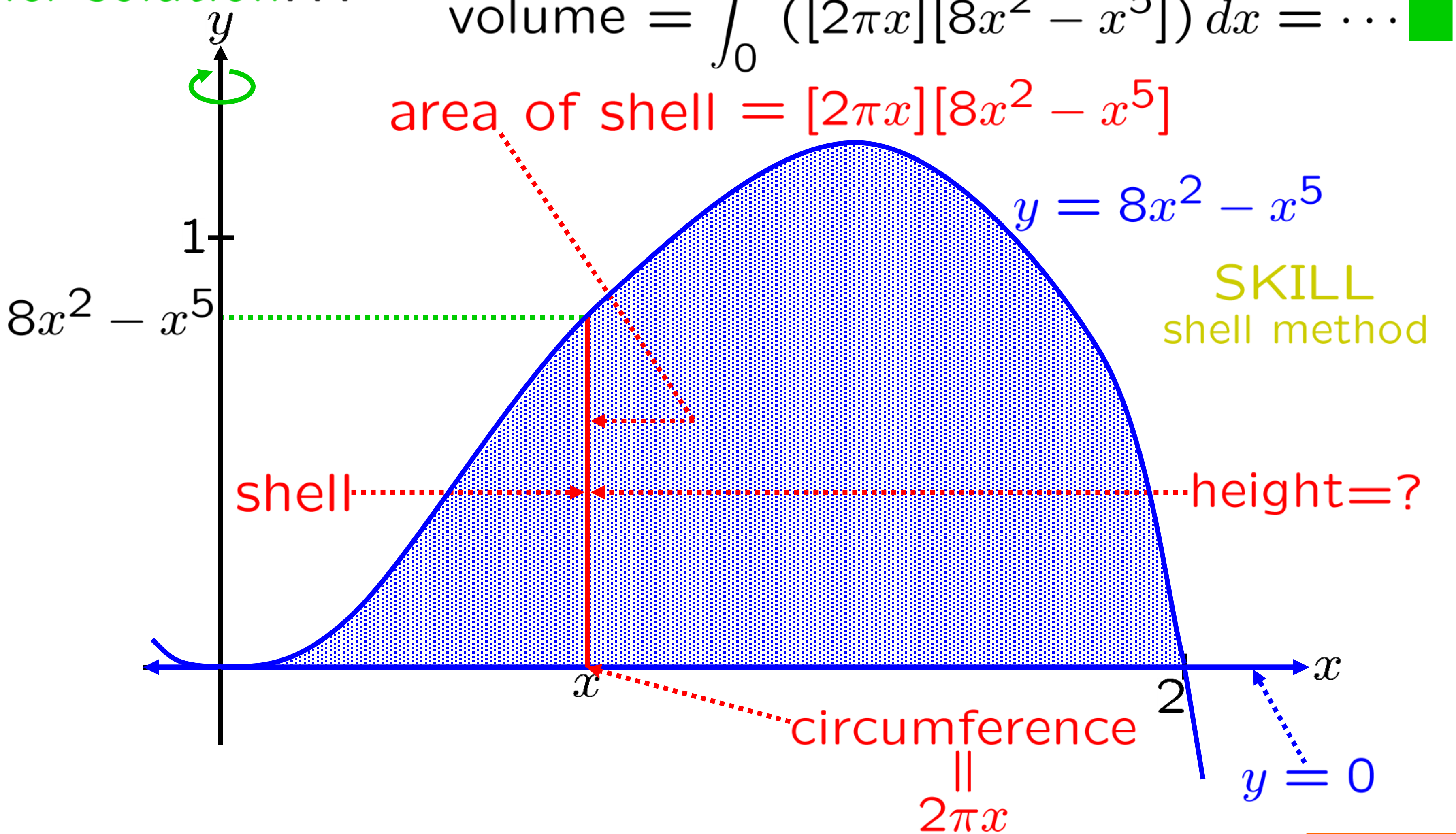


EXAMPLE: Find the volume of the solid formed by revolving, about the y-axis, the region bounded by $y = 8x^2 - x^5$ and $y = 0$.

Easier solution...

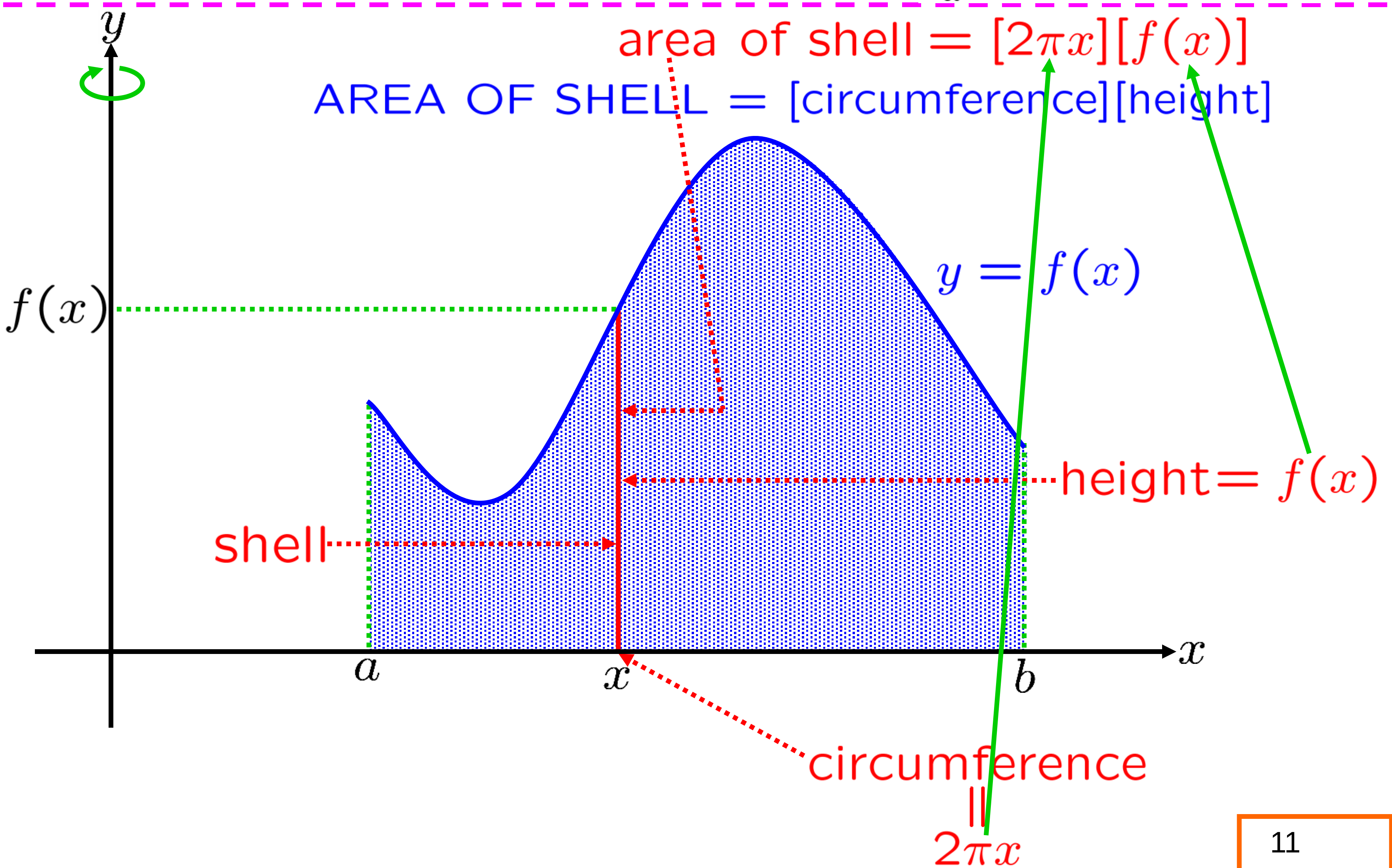
$$\text{volume} = \int_0^2 ([2\pi x][8x^2 - x^5])\, dx = \cdots \blacksquare$$



Next: General formulas...

Say $f \geq 0$ on $[a, b]$. The volume of the solid formed by revolving, about the y-axis, the region under the curve $y = f(x)$ from $x = a$ to $x = b$, is $\displaystyle\int_a^b [\boxed{2\pi} x][f(x)]\,dx$.

if desired

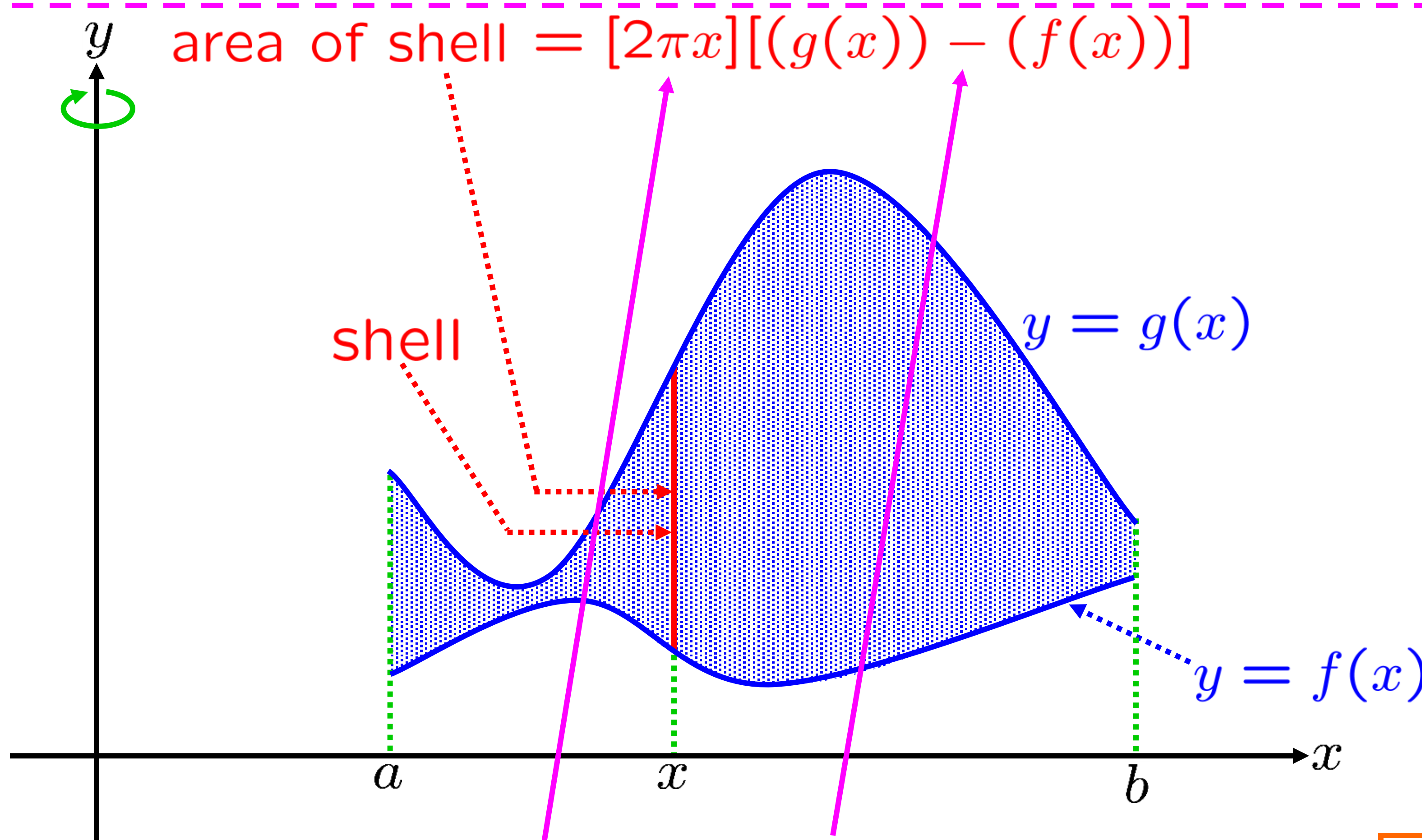


VARIATION: Suppose $f(x) \le g(x)$, for $a \le x \le b$.

The volume of the solid formed by revolving, about the y-axis, the region between $y = f(x)$ and $y = g(x)$

if desired

from $x = a$ to $x = b$, is $\displaystyle\int_a^b [\boxed{2\pi} x][(g(x)) - (f(x))]\,dx$.



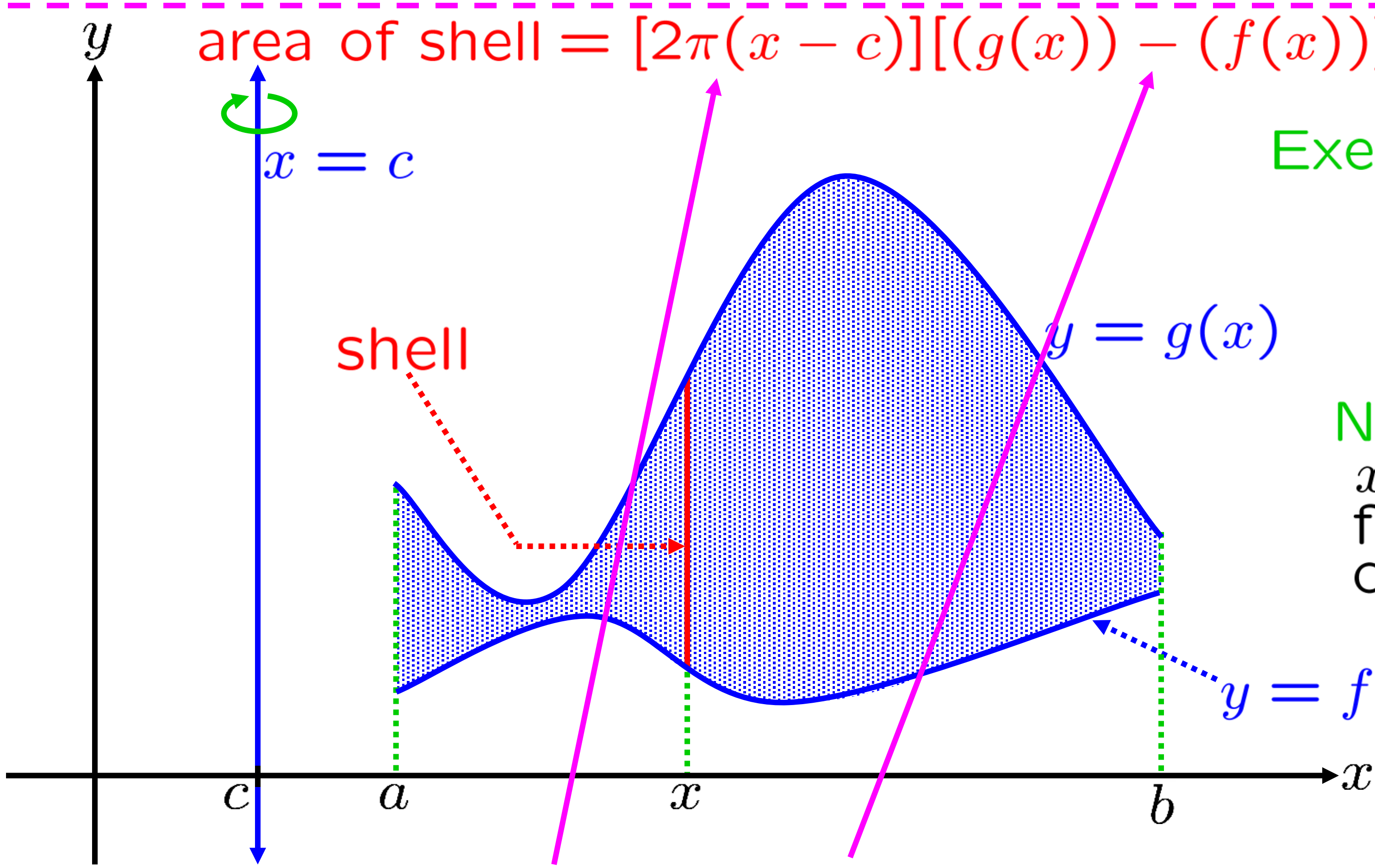
AREA OF SHELL = [circumference][height]

Say $c < a$. VARIATION: Suppose $f(x) \le g(x)$, for $a \le x \le b$.

The volume of the solid formed by revolving, about the $x = c$, the region between $y = f(x)$ and $y = g(x)$

if desired

from $x = a$ to $x = b$, is $\int_a^b [\boxed{2\pi}(x-c)][(g(x)) - (f(x))]\,dx$



Exercise: $c > b$

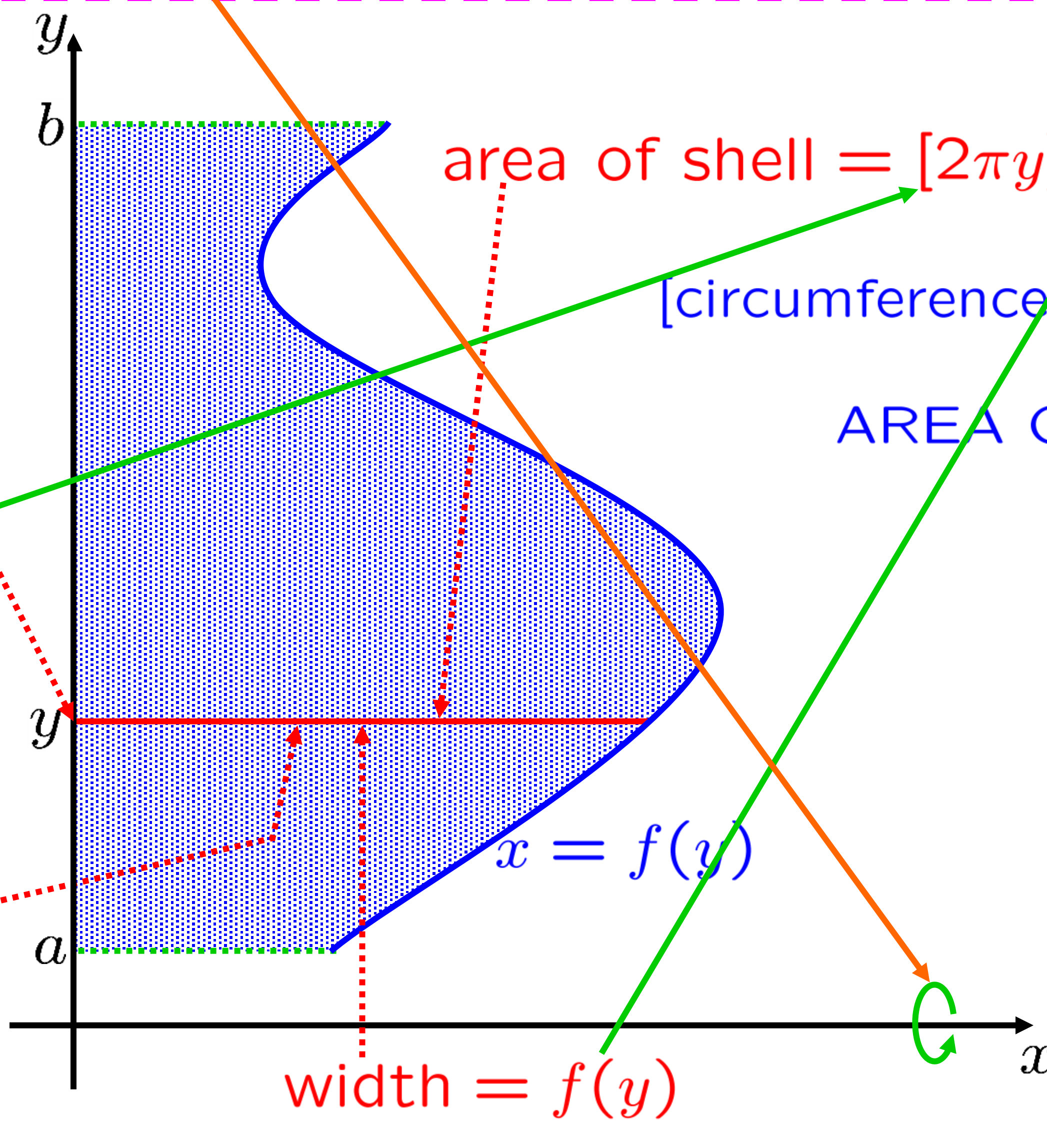
Next: x as a function of y...

AREA OF SHELL = [circumference][height]

§9.3

Say $f \geq 0$ on $[a, b]$. The vol. of the solid formed by revolving, about the x-axis, the region between the y-axis and the curve $\underline{x = f(y)}$ from $y = a$ to $y = b$, is $\displaystyle\int_a^b [\boxed{2\pi} y][f(y)]\, dy$.

if desired



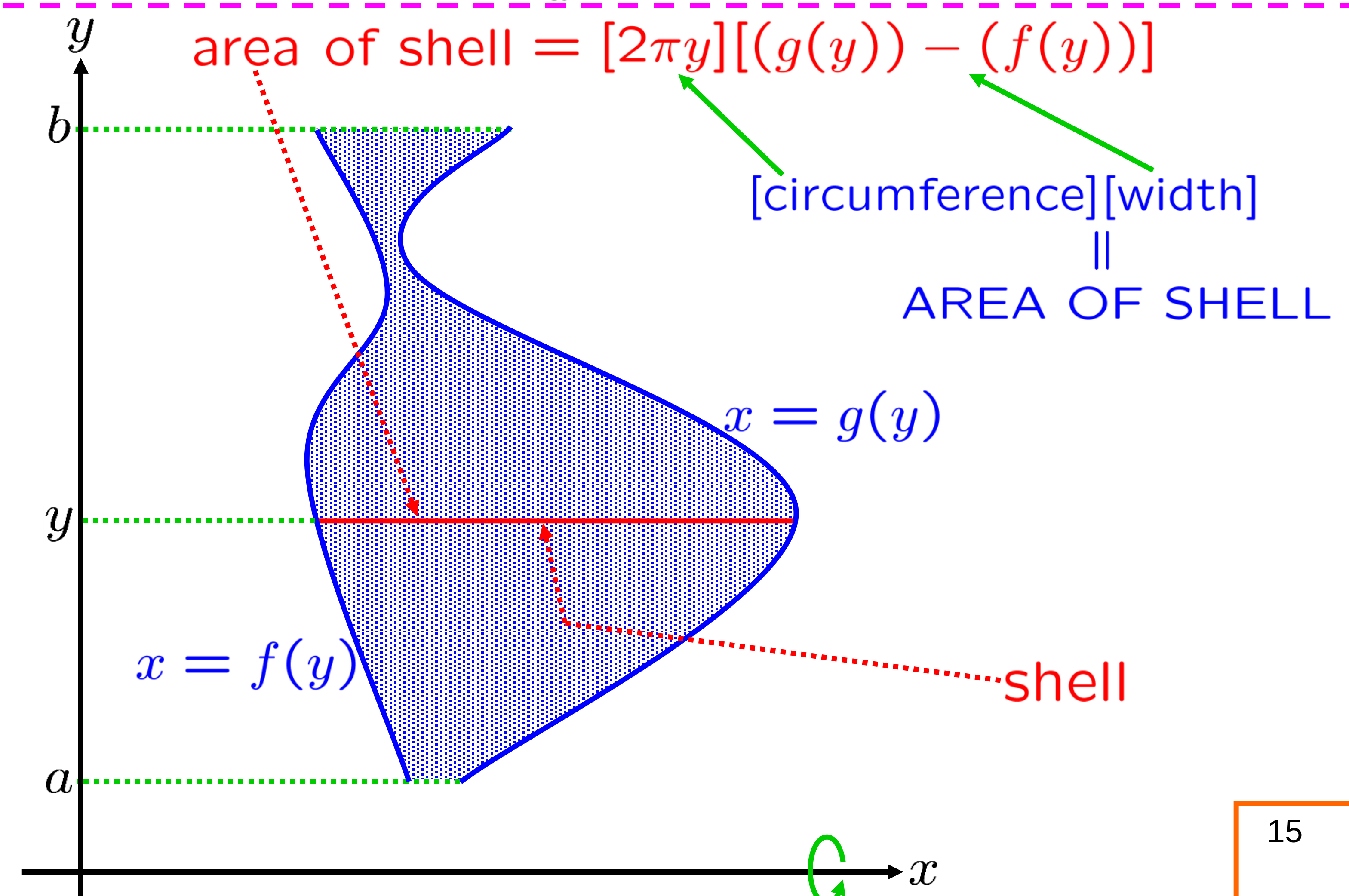
Next:
x as a function of y. . .

VARIATION: Suppose $f(y) \leq g(y)$, for $a \leq y \leq b$.

The volume of the solid formed by revolving, about the x-axis, the region between $x = f(y)$ and $x = g(y)$ from $y = a$ to $y = b$, is

if desired

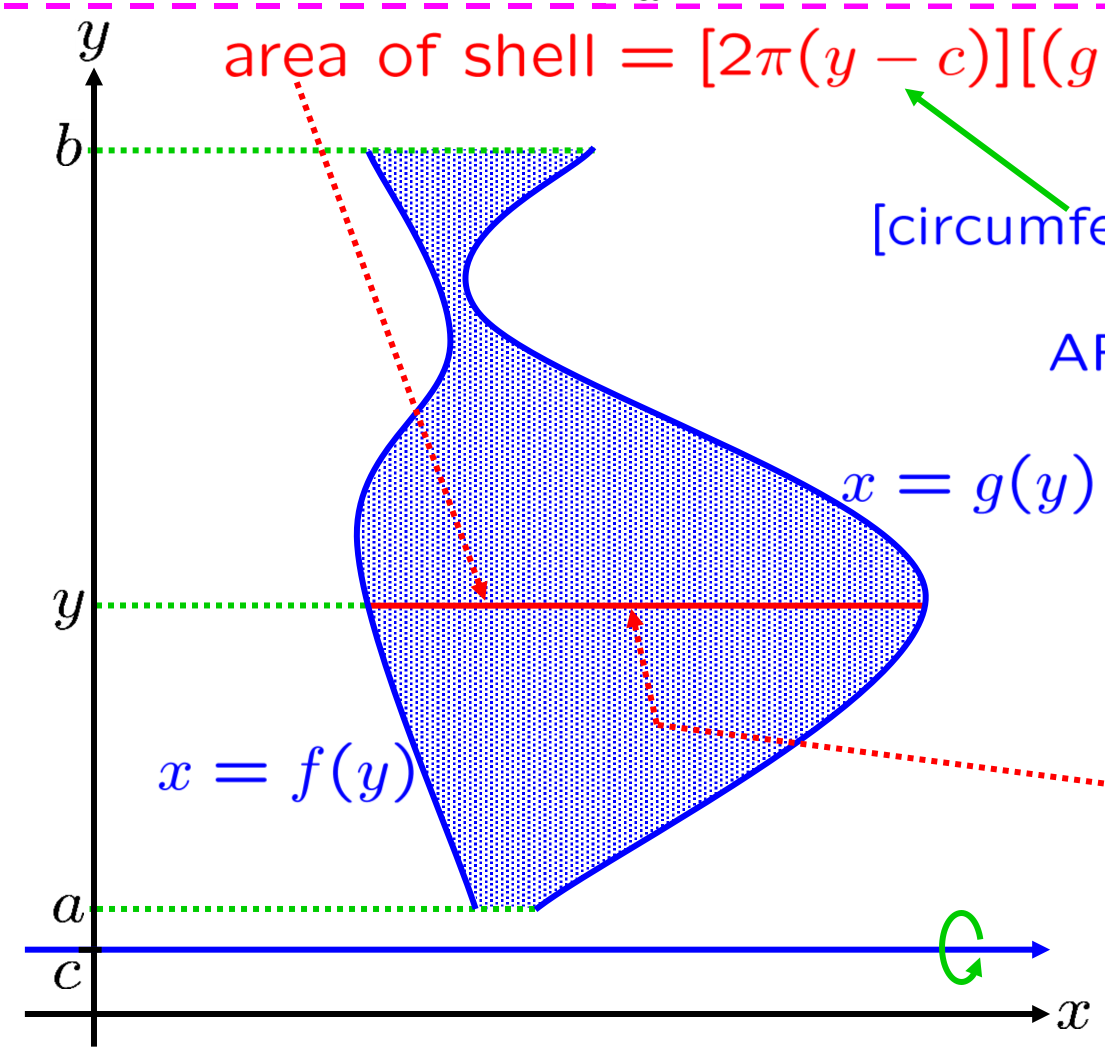
$$\int_a^b [\boxed{2\pi} y][(g(y)) - (f(y))]\, dy.$$



Say $c < a$. VARIATION: Suppose $f(y) \leq g(y)$, for $a \leq y \leq b$. The volume of the solid formed by revolving, about the $y = c$, the region between $x = f(y)$ and $x = g(y)$ from $y = a$ to $y = b$, is

if desired

$$\int_a^b [\boxed{2\pi}(y - c)][(g(y)) - (f(y))]\, dy.$$



Exercise: $c > b$

STOP