4603 HW1

1. Give examples of the following:
    (i) A function which is bijective;
    (ii) A function which is neither injective nor surjective;
    (iii) A function which is injective but not surjective;
    (iv) A function which is surjective but not injective.

2. Let $f : S \to T$ be a function and $A \subset S$. Prove that $A \subset f^{-1}(f(A))$ and that $A = f^{-1}(f(A))$ if f is injective.

3. Let $f : S \to T$ be a function and $B \subset T$. Prove that $B \supset f(f^{-1}(B))$ and that $B = f(f^{-1}(B))$ if f is surjective.

4. Use the well-ordering principle to prove the principle of strong induction.

5. Use strong induction to prove that every natural number can be written as a sum of distinct powers of two.