4603 HW11

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(0) = f'(0) = 0$ and $f''(x) - f(x) = 0$ for all $x \in \mathbb{R}$. Use Taylor's theorem to show that $f(x) = 0$ for all $x \in \mathbb{R}$.

2. Define $f : [0, 1] \to \mathbb{R}$ by $f(x) = x$ if $x \in \mathbb{Q}$ and $f(x) = 1 - x$ if $x \notin \mathbb{Q}$. Prove that f is not Riemann integrable.

3. Let $f : [a, b] \to \mathbb{R}$ be a continuous, nonnegative function. Suppose $f(c) > 0$ for some $c \in [a, b]$. Prove that then $\int_a^b f\,dx > 0$.

4. Define $f : [0, 1] \to \mathbb{R}$ by $f(x) = x^2$. Prove that f is Riemann integrable and $\int_0^1 f\,dx = 1/3$. You may use without justification the formula $1^2 + 2^2 + \ldots + n^2 = n(n + 1)(2n + 1)/6$.