4603 HW3

1. Prove that Cauchy sequences are bounded.

2. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers such that $\{a_n : n \in \mathbb{N}\}$ has exactly one accumulation point, L. Must it be true that $\lim a_n = L$? Prove it, or give a counterexample.

3. Let $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$ for $n \geq 1$. Prove that $\lim a_n = 2$.

4. If S is a set of real numbers, let A_S be the set of all accumulation points of S. Give an example of a bounded countably infinite set $S \subset \mathbb{R}$ such that A_S is countably infinite.

5. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers such that $a_n \neq 0$ for all n and $\lim a_n = L$, where $L \neq 0$. Prove that $\lim a_n^{-1} = L^{-1}$.