4603 HW4

1. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers. Prove that if x is an accumulation point of $\{a_n : n \in \mathbb{N}\}$, then $\{a_n\}_{n=1}^{\infty}$ has a subsequence converging to x.

2. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = 0$ if $x < 0$ and $f(x) = 1$ if $x \geq 0$. Show that f has no limit at 0 in the following ways:

(i) By using the $\epsilon - \delta$ definition of limits of functions;

(ii) By using the sequential definition of limits of functions.

3. Prove that $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ for any $a > 0$.