

4603 HW7

1. If $f + g$ is continuous at a , must it be true that both f and g are continuous at a ? Prove it, or provide a counterexample.
2. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = x^2$. Prove that f is continuous.
3. Define $f : (0, 1) \rightarrow \mathbb{R}$ by $f(x) = 1/x$. Prove that f is continuous but not uniformly continuous.
4. Prove that if $E \subset \mathbb{R}$ is nonempty, closed and bounded above, then $\sup E$ is an element of E .
5. Prove the following statements about open sets in \mathbb{R} :
 - (i) Any union of open sets is open.
 - (ii) A finite intersection of open sets is open.
 - (iii) A set is open if and only if it is a union of open intervals.