MATH 4603 Fall 2022, Final Exam
Handout date: Tuesday 13 December 2022
Due: Tuesday 20 December 2022 at 11:59pm
Instructor: Scot Adams

PRINT YOUR NAME:

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NO COLLABORATION.

Hand-graded problems. Show work.

1. (20 pts) Show:
Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, $k \in \mathbb{N}_0$. Assume: $(f_0 = 0) \,\&\, (f' \in \mathcal{o}(k))$.
Then: $f \in \mathcal{o}(k+1)$.

2. (20 pts) Show:
Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$. Assume $f_0 = f_0' = f_0'' = 0$. Then: $f \in \wp(2)$.

3. (20 pts) Show:
Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, $a \in \mathbb{D}''_f$. Assume: $(f'_a = 0) \,\&\, (f''_a > 0)$.
Then: f has a local strict-minimum at a.

4. (20 pts) Show:
Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$. Assume f is one-to-one. Let $g := f^{-1}$.
Assume: $(\, f_0 = 0 \,) \,\&\, (\, f_0' = 3 \,) \,\&\, (\, g \in \mathcal{O}(0) \,)$. Then: $g_0' = 1/3$.

5. (20 pts) Show:
Let $a \in \mathbb{R}$, $b \geqslant a$, $I := [a; b]$, $f, g : \mathbb{R} \dashrightarrow \mathbb{R}$. Assume: $I \subseteq \mathbb{D}_f \bigcap \mathbb{D}_g$.
Assume: on I, $f \leqslant g$. Then: $\mathrm{JL}_I f \leqslant \mathrm{JL}_I g$.