MATH 4603 Fall 2022, Midterm #1
Handout date: Thursday 13 October 2022
Instructor: Scot Adams

PRINT YOUR NAME:

PRINT YOUR @umn.edu EMAIL ADDRESS:

Thirty minute exam.

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let f be a function. By f is **1-1**, we mean: $\ldots$

B. (5 pts) Let f be an injective function.
Then $f^{-1} : \mathbb{I}_f \to \mathbb{D}_f$ is defined by: $\forall y \in \mathbb{I}_f,\ f_y^{-1} = \cdots$.

C. (5 pts) Let $S \subseteq \mathbb{R}^*$. Then $\mathrm{UB}_S := \cdots$.

D. (5 pts) Let $S \subseteq \mathbb{R}^*$. Then $\sup_S := \cdots$.

E. (5 pts) Let f and g be functions.
Then $g \circ f$ is the function defined by: $\forall x$, $(g \circ f)_x = \cdots$.

F. (5 pts) Let A be a set.
Then $\mathrm{id}^A : A \to A$ is the function defined by: $\forall x \in A$, $\mathrm{id}^A_x = \cdots$.

II. True or false (no partial credit):

a. (5 pts) Let A, B be sets, $f : A \to B$, $g : B \to A$.
Assume $g \circ f = \mathrm{id}^A$. Then $f : A \hookrightarrow B$.

b. (5 pts) $\min \mathbb{Z} = ☹$.

c. (5 pts) $\sup \varnothing = ☹$.

d. (5 pts) Let A, B and C be sets. Let $f : A \to> B$ and $g : B \to> C$.
Then $g \circ f : A \to> C$.

e. (5 pts) $\forall x, y \in \mathbb{R}$, $\Big| \, |y| \; - \; |x| \, \Big| \; \leqslant \; \Big| \, y \; - \; x \, \Big|$.

III. Hand-graded problems. Show work.

1. (10 pts) Let $S := \{2, 4, 6, 8, \ldots\}$. Compute: $\max S$ and $\sup S$.

(No need to prove that your answers are correct; just giving the correct answers will earn you full credit. However, if you give an incorrect answer, an explanation might be worth some partial credit.)

2. (10 pts) Using the 0-Principle of Mathematical Induction,

show: $\forall j \in \mathbb{N}_0, \quad \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^j} = 2 - \frac{1}{2^j}.$

3. (10 pts) Show: $\forall \varepsilon > 0$, $\exists \delta > 0$ s.t. $\delta^5 + 3\delta^3 - 2\delta < \varepsilon$.

4. (15 pts) Show: $\forall \varepsilon > 0$, $\exists j \in \mathbb{N}$ s.t. $(1/\sqrt[3]{j}) + (1/2^j) < \varepsilon$.