MATH 4603 Fall 2022, Midterm #2
Handout date: Thursday 3 November 2022
Instructor: Scot Adams

PRINT YOUR NAME:

PRINT YOUR @umn.edu EMAIL ADDRESS:

Thirty minute exam.

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ and $a \in \mathbb{R}$.
Then $f_a^{\mathbb{T}} : \mathbb{R} \dashrightarrow \mathbb{R}$ is defined by: $\forall h \in \mathbb{R}$, $(f_a^{\mathbb{T}})_h = \cdots$.

B. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$.
By f is **uniformly continuous**, we mean: ...

C. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, $S \subseteq \mathbb{R}$.
By f is **continuous** on S, we mean: ...

D. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, $a, z \in \mathbb{R}$.
By as $x \to a$, $f_x \to z$, we mean:

E. (5 pts) Let S be a set. By S is **countably infinite**, we mean: ...

F. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$. By f is **Lipschitz**, we mean: ...

II. True or false (no partial credit):

a. (5 pts) $\forall x \in \mathbb{R}, \quad (x^2, x^2, x^2, \ldots) \to x^2.$

b. (5 pts) Let $p, q \in \mathbb{R}$. Assume $p \neq q$.
Then: $\exists \varepsilon > 0$ s.t. $(B(p, \varepsilon)) \cap (B(q, \varepsilon)) = \varnothing$.

c. (5 pts) $\exists 2^{\mathbb{R}} \hookrightarrow \mathbb{R}$.

d. (5 pts) Let $s, t \in \mathbb{R}^{\mathbb{N}}$, $a, b \in \mathbb{R}$.
Assume $s \to a$ and $t \to b$. Then $s \cdot t + s + t \to a \cdot b + a + b$.

e. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$. Assume f is continuous. Then f is Lipschitz.

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PLEASE DO NOT WRITE BELOW THE LINE

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Find $f, g : \mathbb{R} \to \mathbb{R}$ s.t.

$$\begin{array}{lll} \text{as } x \to 1, & f_x \to 5 & \text{and} \\ \text{as } y \to 5, & g_y \to 9 & \text{and} \\ \text{as } x \to 1, & (g \circ f)_x \to 0. & \end{array}$$

(No need to prove that your answer is correct; just giving the correct answer will earn you full credit. However, if you give an incorrect answer, an explanation might be worth some partial credit.)

(NOTE: Avoid unbound variables. To bind f and g, say,

"Define $f : \mathbb{R} \to \mathbb{R}$ by: $\forall x \in \mathbb{R}$, $f_x = \cdots$"

and

"Define $g : \mathbb{R} \to \mathbb{R}$ by: $\forall y \in \mathbb{R}$, $g_y = \cdots$" .)

2. (10 pts) Show: Let $s, t \in \mathbb{R}^{\mathbb{N}}$, $a \in \mathbb{R}$.
Assume $s \to -\infty$ and $t \to \infty$. Then $s - t \to -\infty$.

3. (10 pts) Show: Define $f : \mathbb{R} \to \mathbb{R}$ by: $\forall x \in \mathbb{R},\ f_x = |x|$.
Then f is uniformly continuous.

4. (15 pts) Show: Let $s \in \mathbb{R}^{\mathbb{N}}$.
Assume $s \to -\infty$. Then: $s^2 + s \to \infty$.