Sample Quiz Number 4, Math 1251, Scot Adams, Fall 1998

No calculators. 50 minutes. Every problem is worth one point.

Answers: CGFF EETF GEEF FAETTFFT EEFFTTT FFI

1. Let f be a function whose first derivative is given by $f'(x) = \dfrac{x+4}{(x-1)^3}$. Then the *complete* set of intervals on which f is decreasing is

(A) $(-\infty, -4]$ (B) $(-\infty, -4]$ and $(1, \infty)$ (C) $[-4, 1)$

(D) $(-\infty, -3]$ (E) $(-\infty, 1]$ (F) $(-\infty, 4]$

(G) $(-\infty, -4]$ and $[-1, \infty)$ (H) $[-1, 4]$ (I) $(-\infty, 3]$

(J) $(-\infty, -1)$ (K) none of these

2. Let $f(x) = x^3 + 3x^2 - 9x + 8$. Then

(A) f is increasing on $[0, \infty)$ and is decreasing on $(-\infty, 0]$.

(B) f is increasing on $(-\infty, 0]$ and on $[4, \infty)$ and is decreasing on $[0, 4]$.

(C) f is increasing on $(-\infty, 0)$ and is decreasing on $[0, \infty)$.

(D) f is increasing on $[4, \infty)$ and is decreasing on $(-\infty, 4]$.

(E) f is increasing everywhere.

(F) f is decreasing on $(-\infty, -3]$ and on $[1, \infty)$ and is increasing on $[-3, 1]$.

(G) f is increasing on $(-\infty, -3]$ and on $[1, \infty)$ and is decreasing on $[-3, 1]$.

(H) f is decreasing everywhere.

(I) none of these.

3. Let $f(x) = |x-1|$. Then there exists a number $c \in (0, 2)$ such that

$$f(2) - f(0) = [f'(c)][2-0].$$

(T) True (F) False

4. Let $f(x) = (x+2)^5$. Then f has a point of inflection at the point $(2, 0)$.

(T) True (F) False

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5. Let $f(x) = x^7$. Then

(A) f is concave upward on $(-\infty, -7]$ and is concave downward on $[-7, \infty)$.

(B) f is concave downward on $(-\infty, -7]$ and is concave upward on $[-7, \infty)$.

(C) f is concave upward on $(-\infty, -1]$ and on $[1, \infty)$ and is concave downward on $[-1, 1]$.

(D) f is concave downward on $(-\infty, -1]$ and on $[1, \infty)$ and is concave upward on $[-1, 1]$.

(E) f is concave downward on $(-\infty, 0]$ and is concave upward on $[0, \infty)$.

(F) f is concave upward on $(-\infty, 0]$ and is concave downward on $[0, \infty)$.

(G) f is concave upward everywhere.

(H) f is concave downward everywhere.

6. Let $f(x) = 16x^3 + 12x^2 + 6x + 5$. Then

(A) f is concave upward on $(-\infty, -1/4]$ and is concave downward on $[-1/4, \infty)$.

(B) f is concave upward on $[0, \infty)$ and is concave downward on $(-\infty, 0]$.

(C) f is concave upward on $(-\infty, 1/4]$ and is concave downward on $[1/4, \infty)$.

(D) f is concave downward on $(-\infty, 1/4]$ and is concave upward on $[1/4, \infty)$.

(E) f is concave downward on $(-\infty, -1/4]$ and is concave upward on $[-1/4, \infty)$.

(F) f is concave upward on $(-\infty, 0]$ and is concave downward on $[0, \infty)$.

(G) f is concave upward on $[2, \infty)$ and is concave downward on $(-\infty, 2]$.

(H) f is concave upward on $(-\infty, 2]$ and is concave downward on $[2, \infty)$.

(I) f is concave upward everywhere.

7. Let f be twice differentiable on $(-1, 1)$ and assume that $f''(x) > 0$, for all $x \in (-1, 1)$. Then f is concave upward on $(-1, 1)$.

(T) True (F) False

8. If a function is increasing on an interval, then it must be concave upward on that interval.

(T) True (F) False

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9. Let $f(x) = x^4 + 4x^3 + 4x^2 + 8$. Then

(A) f is increasing on $[-\sqrt{3}, 0]$ and on $[\sqrt{3}, \infty)$ and is decreasing on $(-\infty, -\sqrt{3}]$ and on $[0, \sqrt{3}]$.

(B) f is decreasing on $(-\infty, 0]$ and is increasing on $[0, \infty)$.

(C) f is increasing on $(-\infty, -\sqrt{3}]$ and on $[0, \sqrt{3}]$ and is decreasing on $[-\sqrt{3}, 0]$ and on $[\sqrt{3}, \infty)$.

(D) f is increasing everywhere.

(E) f is decreasing everywhere.

(F) f is increasing on $(-\infty, -2]$ and on $[-1, 0]$ and is decreasing on $[-2, -1]$ and on $[0, \infty)$.

(G) f is decreasing on $(-\infty, -2]$ and on $[-1, 0]$ and is increasing on $[-2, -1]$ and on $[0, \infty)$.

(H) none of these.

10. The illumination of an object by a light source is equal to

$$\frac{\text{the strength of the source}}{(\text{the distance from the source to the object})^2}.$$

Imagine that two light sources are placed on the real number line. One is of strength one and is placed at 0. The other is of strength eight and is placed at 15. The object is placed between them. Let x denote the number at which the object should be placed so as to receive the *least* illumination. Then x is equal to

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6 (G) 10 (H) 11 (I) 12 (J) 13 (K) 14

(L) none of these

11. A farmer with 1200 feet of fencing wants to enclose a rectangular area and then divide it into two pens with fencing parallel to one side of the rectangle. What is the largest possible total area (in square feet) of the two pens?

(A) (150)(150) (B) (50)(450) (C) (120)(290) (D) (100)(300)

(E) none of these

12. The graph of $y = \arcsin x$ has no points of inflection.

(T) True (F) False

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13. The graph of $y = 2x^2 + 3x + 5$ is increasing on $(-\infty, \infty)$.

(T) True (F) False

14. Find the critical numbers of $f(x) = xe^{4x}$.

(A) $-1/4$ only (B) 0 and 1 (C) $-1/4$ and 0 (D) $-1/4$ and 1

(E) none of these

15. Find the absolute minimum and absolute maximum values of $f(x) = x^2 - 2x + 2$ on $[0, 2]$.

(A) 1 and 5 (B) 1 and 10 (C) 5 and 10 (D) 2 and 5

(E) 1 and 2 (F) 1 and 3 (G) 0 and 4 (H) 0 and 5

(I) none of these

16. If a function is continuous on the interval $[2, 3]$, then it must attain a maximum value on $[2, 3]$.

(T) True (F) False

17. Let f be a function whose domain is the interval $[1, 5]$. Suppose that f attains an absolute maximum at 3. Then 3 must be a critical number for f.

(T) True (F) False

18. True or False: Assume that f is a function which is differentiable at every real number. Assume also that $f'(3) = 0$. Then f attains a local extremum at 3.

(T) True (F) False

19. Let x, y, r, θ be defined as in planetary motion. Suppose we prove that $1/r = 2 + 3\sin\theta$. Find A, B, C, D, E, F such that $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ and such that $A = -4$. Then C is equal to

(A) 1 (B) 2 (C) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) none of these

20. True or False: If two functions are both differentiable at every real number, and if they have the same derivative, then they must differ by a constant.

(T) True (F) False

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21. Recall that $\dot{r} = dr/dt$ and that $\dot{\theta} = d\theta/dt$. Compute $[d/dt][r \csc\theta]$.

(A) $\dot{r} \csc \dot{\theta}$ (B) $\dot{r} \csc\theta \cot\theta$ (C) $\dot{r} \csc\theta + r\dot{\theta} \csc\theta \cot\theta$

(D) $-\dot{r} \csc\theta + r\dot{\theta} \csc\theta \cot\theta$ (E) $\dot{r} \csc\theta - r\dot{\theta} \csc\theta \cot\theta$ (F) none of these

22. Find the intervals of increase and decrease for $f(x) = x^3$.

(A) f is increasing on $(-\infty, 0]$ and is decreasing on $[0, \infty)$.

(B) f is decreasing on $(-\infty, 0]$ and is increasing on $[0, \infty)$.

(C) f is increasing on $(-\infty, -1]$ and on $[1, \infty)$ and is decreasing on $[-1, 1]$;

(D) f is decreasing on $(-\infty, -1]$ and on $[1, \infty)$ and is increasing on $[-1, 1]$;

(E) f is increasing everywhere.

(F) f is decreasing everywhere.

(G) none of these.

23. True or False: A local maximum is always an absolute maximum.

(T) True (F) False

24. True or False: An absolute maximum is always a local maximum.

(T) True (F) False

25. True or False: If f is increasing on $(-\infty, \infty)$, then f is one-to-one.

(T) True (F) False

26. True or False: If f is a constant function, then f has a local maximum at every real number. (NOTE: The domain of any constant function is the set of all real numbers.)

(T) True (F) False

27. True or False: Let f be continuous on the compact interval $[a, b]$. Then f attains an absolute maximum on $[a, b]$ and an absolute minimum on $[a, b]$.

(T) True (F) False

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28. True or False: Let f be continuous on the open interval (a, b). Then f attains an absolute maximum and an absolute minimum on (a, b).

(T) True (F) False

29. True or False: A constant function is decreasing.

(T) True (F) False

30. Compute the components of the gravitational force arrow for a particle of mass 50 located at the point $(3, -4)$.

(A) $\langle 3, -4\rangle$ (B) $\langle 3/50, -4/50\rangle$ (C) $\langle -3, 4\rangle$

(D) $\langle -3/50, 4/50\rangle$ (E) $\langle 6, -8\rangle$ (F) $\langle -6, 8\rangle$

(G) $\langle 3/5, 4/5\rangle$ (H) $\langle -3/5, 4/5\rangle$ (I) none of these

END OF TEST