START

Householder Method

Problem: $A := \begin{bmatrix} 9 & 6 & 8 \\ 2 & 1 & -1 \\ 6 & -2 & -8 \end{bmatrix}$.

Find orthog $Q \in \mathbb{R}^{3\times 3}$, UT $R \in \mathbb{R}^{3\times 3}$ with pos. diag entries s.t. $A = QR$.

Gram-Schmidt

Soln: $A \begin{bmatrix} 1 & -a & -b \\ & 1 & \\ & & 1 \end{bmatrix} = \begin{bmatrix} 9 & 6-9a & 8-9b \\ 2 & 1-2a & -1-2b \\ 6 & -2-6a & -8-6b \end{bmatrix}$.

$$0 = \begin{pmatrix} -54-81a \\ +2-4a \\ -12-36a \end{pmatrix} = 44-121a \therefore a = \frac{44}{121}.$$

$$0 = \begin{pmatrix} 72-81b \\ -2-4b \\ -48-36b \end{pmatrix} = 22-121b \therefore b = \frac{22}{121}.$$

$$A\begin{bmatrix}1 & -a & -b\\ & 1 & \\ & & 1\end{bmatrix} = \begin{bmatrix}9 & 6-9a & 8-9b\\ 2 & 1-2a & -1-2b\\ 6 & -2-6a & -8-6b\end{bmatrix}.$$

$$a=\frac{44}{121}.$$

$$b=\frac{22}{121}.$$

$$A\begin{bmatrix}1 & -a & -b\\ & 1 & \\ & & 1\end{bmatrix} = \begin{bmatrix}9 & \frac{726}{121}-\frac{396}{121} & \frac{968}{121}-\frac{198}{121}\\ 2 & \frac{121}{121}-\frac{88}{121} & -\frac{121}{121}-\frac{44}{121}\\ 6 & -\frac{242}{121}-\frac{264}{121} & -\frac{968}{121}-\frac{132}{121}\end{bmatrix}$$

$$= \begin{bmatrix}9 & \frac{330}{121} & \frac{770}{121}\\ 2 & \frac{33}{121} & -\frac{165}{121}\\ 6 & -\frac{506}{121} & -\frac{1100}{121}\end{bmatrix}.$$

$$\left(\begin{array}{l}2700+270\\ +66\\ -3036=0\end{array}\right) \qquad \overset{!!}{x_1} \qquad \overset{\uparrow}{x_1\perp} \qquad \overset{\uparrow}{x_1\perp} \qquad \left(\begin{array}{l}6300+630\\ -330\\ -6600=0\end{array}\right)$$

$$A \begin{bmatrix} 1 & -a & -b \\ & 1 & \\ & & 1 \end{bmatrix} = \begin{bmatrix} 9 & \frac{330}{121} & \frac{770}{121} \\ 2 & \frac{33}{121} & -\frac{165}{121} \\ 6 & -\frac{506}{121} & -\frac{1100}{121} \end{bmatrix}.$$

$$\begin{array}{ccc} \| & \Uparrow & \Uparrow \\ x_1 & x_1^{\perp} & x_1^{\perp} \end{array}$$

$$A \begin{bmatrix} 1 & -a & -b \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & -c \\ & & 1 \end{bmatrix} = \begin{bmatrix} 9 & \frac{330}{121} & \frac{770}{121} - \frac{330}{121}c \\ 2 & \frac{33}{121} & -\frac{165}{121} - \frac{33}{121}c \\ 6 & -\frac{506}{121} & -\frac{1100}{121} + \frac{506}{121}c \end{bmatrix}.$$

$$\begin{array}{ccc} \| & \Uparrow & \Uparrow \\ x_1 & x_1^{\perp} & x_1^{\perp} \end{array}$$

$$0 = \frac{1}{121^2} \cdot \left(\begin{array}{l} 330 \cdot 770 - 330^2 c \\ -33 \cdot 165 - 33^2 c \\ +506 \cdot 1100 - 506^2 c \end{array} \right) \qquad \therefore \quad c = \frac{330 \cdot 770 - 33 \cdot 165 + 506 \cdot 1100}{330^2 + 33^2 + 506^2}.$$

$$A\begin{bmatrix} 1 & -a & -b \\ & 1 & \\ & & 1 \end{bmatrix}\begin{bmatrix} 1 & & \\ & 1 & -c \\ & & 1 \end{bmatrix} = \begin{bmatrix} 9 & \frac{330}{121} & \frac{770}{121} - \frac{330}{121}c \\ 2 & \frac{33}{121} & -\frac{165}{121} - \frac{33}{121}c \\ 6 & -\frac{506}{121} & -\frac{1100}{121} + \frac{506}{121}c \end{bmatrix}.$$

$$\begin{array}{ccc} \| & \| & \| \\ x_1 & x_2 & x_3 \in x_2^\perp \\ & \cap & \cap \\ & x_1^\perp & x_1^\perp \end{array}$$

$$A = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}\begin{bmatrix} 1 & & \\ & 1 & c \\ & & 1 \end{bmatrix}\begin{bmatrix} 1 & a & b \\ & 1 & \\ & & 1 \end{bmatrix}.$$

pw orthog cols

pw orthog cols

$$A = \begin{bmatrix} & & \\ x_1 & x_2 & x_3 \\ & & \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & c \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & a & b \\ & 1 & \\ & & 1 \end{bmatrix}.$$

$$A = \begin{bmatrix} \frac{x_1}{\|x_1\|} & \frac{x_2}{\|x_2\|} & \frac{x_3}{\|x_3\|} \end{bmatrix} \begin{bmatrix} \|x_1\| & & \\ & \|x_2\| & \\ & & \|x_3\| \end{bmatrix} \begin{bmatrix} 1 & a & b \\ & 1 & c \\ & & 1 \end{bmatrix}.$$

pw on cols

$\therefore$ orthog

$$\underbrace{A = \begin{bmatrix} \frac{x_1}{\|x_1\|} & \frac{x_2}{\|x_2\|} & \frac{x_3}{\|x_3\|} \end{bmatrix}}_{\substack{\shortparallel \\ Q \text{ orthog}}} \underbrace{\begin{bmatrix} \|x_1\| & & \\ & \|x_2\| & \\ & & \|x_3\| \end{bmatrix} \begin{bmatrix} 1 & a & b \\ & 1 & c \\ & & 1 \end{bmatrix}}_{\substack{\shortparallel \\ R \text{ UT with pos diag entries}}}.$$

$$a = \frac{44}{121}.$$

$$b = \frac{22}{121}.$$

$$c = \frac{330 \cdot 770 - 33 \cdot 165 + 506 \cdot 1100}{330^2 + 33^2 + 506^2}.$$

$$\underset{x_1}{\underset{\shortparallel}{\begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix}}} \quad \underset{x_2}{\underset{\shortparallel}{\begin{bmatrix} \frac{330}{121} \\ \frac{33}{121} \\ -\frac{506}{121} \end{bmatrix}}} \quad \underset{x_3}{\underset{\shortparallel}{\begin{bmatrix} \frac{770}{121} - \frac{330}{121}c \\ -\frac{165}{121} - \frac{33}{121}c \\ -\frac{1100}{121} + \frac{506}{121}c \end{bmatrix}}}$$

Householder method ...

$$I_1 = \begin{bmatrix} 1 \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

etc.

Problem: $A := \begin{bmatrix} 9 & 6 & 8 \\ 2 & 1 & -1 \\ 6 & -2 & -8 \end{bmatrix}$.

Find orthog $Q \in \mathbb{R}^{3\times 3}$, UT $R \in \mathbb{R}^{3\times 3}$ with pos. diag entries s.t. $A = QR$.

Idea: Find orthog $Q_1 \in \mathbb{R}^{3\times 3}$ s.t. $Q_1 \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix}$ is a pos mult of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. $\begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}$

reflection

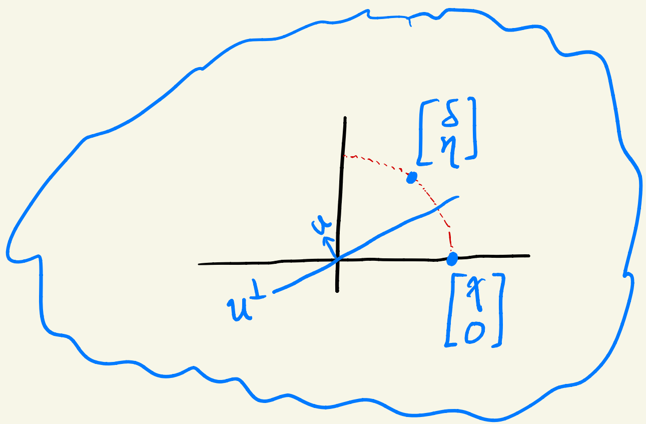
$$A := \begin{bmatrix} 9 & 6 & 8 \\ 2 & 1 & -1 \\ 6 & -2 & -8 \end{bmatrix}.$$

$$Q_1 A = \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \delta & \varepsilon \\ 0 & \eta & \theta \end{bmatrix}.$$

$$Q_1 A = \begin{bmatrix}\alpha & \beta & \gamma \\ 0 & \delta & \varepsilon \\ 0 & \eta & \theta\end{bmatrix}.$$

Find orthog $Q_2 \in \mathbb{R}^{2\times 2}$ s.t. $Q_2 \begin{bmatrix}\delta \\ \eta\end{bmatrix}$ is a pos mult of $\begin{bmatrix}1 \\ 0\end{bmatrix}$. $\begin{bmatrix}\chi \\ 0\end{bmatrix}$

reflection



$$t = \begin{bmatrix}\delta \\ \eta\end{bmatrix} - \begin{bmatrix}\chi \\ 0\end{bmatrix}.$$

$$u = \frac{t}{\|t\|}.$$

$$Q_2 \begin{bmatrix}\delta\\ \eta\end{bmatrix} = \begin{bmatrix}\chi\\ 0\end{bmatrix}.$$

$$Q_2 \begin{bmatrix}\delta & \varepsilon\\ \eta & \theta\end{bmatrix} = \begin{bmatrix}\chi & \psi\\ 0 & \omega\end{bmatrix}.$$

$$Q_1 A = \begin{bmatrix}\alpha & \beta & \gamma\\ 0 & \delta & \varepsilon\\ 0 & \eta & \theta\end{bmatrix}.$$

$I_1 = [1].$

$$\big(\overbrace{I_1 \oplus Q_2}^{\begin{bmatrix}1 & 0 \;\; 0\\ \begin{matrix}0\\ 0\end{matrix} & Q_2\end{bmatrix}}\big) \cdot Q_1 \cdot A = \begin{bmatrix}\alpha & \beta & \gamma\\ 0 & \chi & \psi\\ 0 & 0 & \omega\end{bmatrix}.$$

reflection

$$(I_1 \oplus Q_2) \cdot Q_1 \cdot A = \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \chi & \psi \\ 0 & 0 & \omega \end{bmatrix}.$$

reflection

$$Q_1 \cdot A = (I_1 \oplus Q_2) \cdot \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \chi & \psi \\ 0 & 0 & \omega \end{bmatrix}.$$

$$A = Q_1 \cdot (I_1 \oplus Q_2) \cdot \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \chi & \psi \\ 0 & 0 & \omega \end{bmatrix}.$$

$\alpha, \chi > 0$

later: $\omega < 0$ ̈

$$A = \underbrace{Q_1 \cdot (I_1 \oplus Q_2) \cdot \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \chi & \psi \\ 0 & 0 & -\omega \end{bmatrix}}_{R}.$$

$I_1 = [1]$

$\forall x, y \in \mathbb{R}^{n \times 1}$, $\quad x^T y = (x \odot y) \cdot I_1$.

e.g., $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^T \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = [1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6] = (1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6)[1]$

"inner product"

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}^T = \begin{bmatrix} 1 \cdot 4 & 1 \cdot 5 & 1 \cdot 6 \\ 2 \cdot 4 & 2 \cdot 5 & 2 \cdot 6 \\ 3 \cdot 4 & 3 \cdot 5 & 3 \cdot 6 \end{bmatrix}$$

"outer product"

$\forall x, y \in \mathbb{R}^{n\times 1}$, $\quad x^T y = (x \odot y)\cdot I_1$.

Th Let $u \in \mathbb{R}^{n\times 1}$ be a unit vector.

Let $Q := I_n - 2uu^T$. ("reflection thru $u^\perp$")

Then $(Qu = -u)$ and $(\forall x \in u^\perp, \quad Qx = x)$

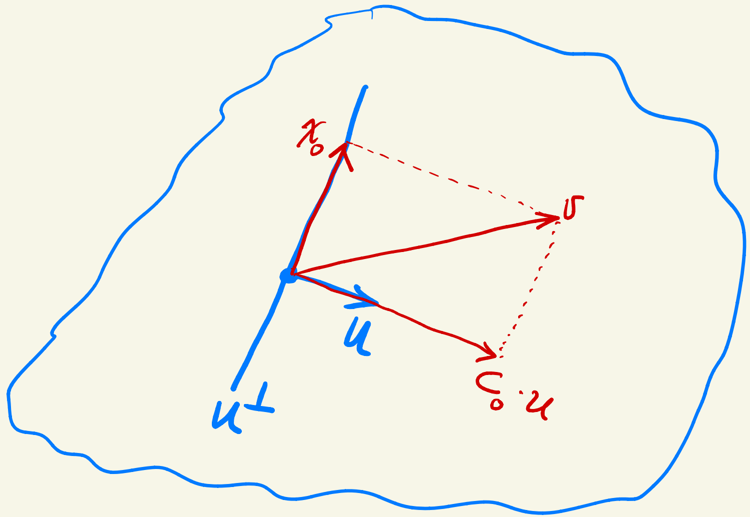
and $(Q^2 = I_n)$ and (Q is orthogonal).

Pf: $Qu = u - 2uu^Tu = u - 2u\cdot 1 \cdot I_1 = u - 2u = -u$.

$\forall x \in u^\perp$, $\quad Qx = x - 2uu^Tx = x - 2u\cdot 0\cdot I_1 = x - 0_{n\times 1} = x$.

Given $v \in \mathbb{R}^{n\times 1}$. Want: $Q^2 v = v$ & $\|Qv\| = \|v\|$.

Given $v \in \mathbb{R}^{n\times 1}$. <u>Want</u>: $Q^2 v = v$ & $\|Qv\| = \|v\|$.



Choose $c_0 \in \mathbb{R}$, $x_0 \in u^\perp$ s.t. $v = c_0 u + x_0$.

$Qu = -u.$ $\qquad \forall x \in u^{\perp}, \quad Qx = x.$

Given $v \in \mathbb{R}^{n \times 1}$. $\quad$ <u>Want</u>: $Q^2 v = v$ & $\|Qv\| = \|v\|.$

$v = c_0 u + x_0. \quad c_0 \in \mathbb{R}. \quad x_0 \in u^{\perp}. \quad Q x_0 = x_0.$

$Qv = -c_0 u + x_0. \qquad Q^2 v = c_0 u + x_0 = v.$

<u>Want</u>: $\|Qv\| = \|v\|.$ $\qquad$ <u>Want</u>: $\|-c_0 u + x_0\| = \|c_0 u + x_0\|.$

$$\|c_0 u + x_0\|^2 = (c_0 u + x_0) \odot (c_0 u + x_0) = c_0^2 + x_0 \odot x_0.$$

$$\|-c_0 u + x_0\|^2 = (-c_0 u + x_0) \odot (-c_0 u + x_0) = c_0^2 + x_0 \odot x_0.$$

$$\|-c_0 u + x_0\|^2 = \|c_0 u + x_0\|^2 \quad \therefore \quad \sqrt{\ } \quad \|-c_0 u + x_0\| = \|c_0 u + x_0\|. \quad \text{QED}$$

$$Q v = w.$$

$$\underbrace{\frac{1}{11}\begin{bmatrix} 9 & 2 & 6 \\ 2 & 9 & -6 \\ 6 & -6 & -7 \end{bmatrix}}_{\text{orthog}} \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix} = \underbrace{\begin{bmatrix} 11 \\ 0 \\ 0 \end{bmatrix}}_{\text{pos mult of } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}.$$

Problem: Let $v := \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix}$, $\alpha := \|v\|$, $w := \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}$.

pos mult of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Find (reflection) orthog $Q_1 \in \mathbb{R}^{3 \times 3}$ s.t. $Q_1 v = w$.

Soln: $\|v\| = \|w\|$. $\quad (v+w) \odot (v-w) = \|v\|^2 - \|w\|^2 = 0$.

$t := v - w$. $\quad x_0 := v + w \in t^\perp$.

$$\left.\begin{aligned} v &= (t + x_0)/2. \\ w &= (-t + x_0)/2. \end{aligned}\right\} \left.\begin{aligned} c_0 &:= \|t\|. \\ u &:= t/c_0. \\ t &= c_0 u. \end{aligned}\right\} \left.\begin{aligned} v &= (c_0 u + x_0)/2. \\ w &= (-c_0 u + x_0)/2. \end{aligned}\right\} x_0 \in u^\perp.$$

$$v = (c_0 u + x_0)/2.$$
$$w = (-c_0 u + x_0)/2.$$

$x_0 \in u^\perp$.

Find orthog $Q_1 \in \mathbb{R}^{3\times 3}$ s.t. $Q_1 v = w$.

$$Q_1 := I_3 - 2uu^T.$$

$Q_1 u = -u$.

Q_1 is orthogonal. (reflection)

$Q_1 x_0 = x_0$.

$$Q_1 v = Q_1 \cdot (c_0 u + x_0)/2 = (-c_0 u + x_0)/2 = w.$$

$v := \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix}$, $\quad \alpha := \|v\|$, $\quad w := \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}$.

$$\alpha = \sqrt{81+4+36} = \sqrt{121} = 11. \qquad w = \begin{bmatrix} 11 \\ 0 \\ 0 \end{bmatrix}.$$

$t := v - w.$

$$t = \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix} - \begin{bmatrix} 11 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 6 \end{bmatrix} = 2 \cdot \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

$c_0 := \|t\|.$

$$c_0 = 2 \cdot \sqrt{1+1+9} = 2 \cdot \sqrt{11}.$$

$u = t / c_0$

$$u = \frac{1}{\sqrt{11}} \cdot \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}.$$

$$Q_1 := I_3 - 2uu^T.$$

$$u = \frac{1}{\sqrt{11}} \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}. \qquad Q_1 := I_3 - 2uu^T.$$

$$uu^T = \frac{1}{11} \begin{bmatrix} 1 & -1 & -3 \\ -1 & 1 & 3 \\ -3 & 3 & 9 \end{bmatrix}.$$

$$Q_1 = \frac{1}{11} \begin{bmatrix} 11 & & \\ & 11 & \\ & & 11 \end{bmatrix} - \frac{1}{11} \begin{bmatrix} 2 & -2 & -6 \\ -2 & 2 & 6 \\ -6 & 6 & 18 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 9 & 2 & 6 \\ 2 & 9 & -6 \\ 6 & -6 & -7 \end{bmatrix}.$$

$$\underbrace{\frac{1}{11}\begin{bmatrix} 9 & 2 & 6 \\ 2 & 9 & -6 \\ 6 & -6 & -7 \end{bmatrix}}_{\text{orthog}} \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix} = \underbrace{\begin{bmatrix} 11 \\ 0 \\ 0 \end{bmatrix}}_{\text{pos mult of } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}.$$

Problem: $A := \begin{bmatrix} 9 & 6 & 8 \\ 2 & 1 & -1 \\ 6 & -2 & -8 \end{bmatrix}.$

Find orthog $Q \in \mathbb{R}^{3\times 3}$, UT $R \in \mathbb{R}^{3\times 3}$ with pos. diag entries

s.t. $A = QR$.

Householder

Soln: $\frac{1}{11}\begin{bmatrix} 9 & 2 & 6 \\ 2 & 9 & -6 \\ 6 & -6 & -7 \end{bmatrix}\begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 11 \\ 0 \\ 0 \end{bmatrix}.$

$Q_1 := \frac{1}{11}\begin{bmatrix} 9 & 2 & 6 \\ 2 & 9 & -6 \\ 6 & -6 & -7 \end{bmatrix}.$ $\qquad Q_1 A = \begin{bmatrix} 11 & 4 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 10 \end{bmatrix}.$

$$Q_1 A = \begin{bmatrix} 11 & 4 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 10 \end{bmatrix}.$$

$$\left\| \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\| = \sqrt{9+16} = \sqrt{25} = 5$$

<u>Subgoal</u>: Find orthog $Q_2 \in \mathbb{R}^{2\times 2}$ s.t. $Q_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$.

Solved later.

<u>Soln</u>: $Q_2 = \frac{1}{5}\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$.

$$Q_3 = I_1 \oplus Q_2 = \frac{1}{5}\begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & -3 \end{bmatrix}. \qquad Q_3 Q_1 A = \underbrace{\begin{bmatrix} 11 & 4 & 2 \\ 0 & 5 & 11 \\ 0 & 0 & -2 \end{bmatrix}}_{UT}.$$

$I_1 = [1]$.

$$Q_3 Q_1 A = \begin{bmatrix} 11 & 4 & 2 \\ 0 & 5 & 11 \\ 0 & 0 & -2 \end{bmatrix}.$$

$$Q_1^2 = I_3 = Q_2^2.$$

$$A = Q_1 Q_3 \begin{bmatrix} 11 & 4 & 2 \\ 0 & 5 & 11 \\ 0 & 0 & -2 \end{bmatrix}$$

$$= \underbrace{Q_1 Q_3 \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix}}_{Q \text{ orthog}} \underbrace{\begin{bmatrix} 11 & 4 & 2 \\ 0 & 5 & 11 \\ 0 & 0 & 2 \end{bmatrix}}_{R \text{ UT with pos diag entries}}.$$

<u>Subgoal</u>: Find orthog $Q_2 \in \mathbb{R}^{2\times 2}$ s.t. $Q_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$.

<u>Soln</u>: $v := \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $w := \begin{bmatrix} 5 \\ 0 \end{bmatrix}$. $\|v\| = \|w\|$.

$$(v+w) \odot (v-w) = \|v\|^2 - \|w\|^2 = 0.$$

$$t := \frac{v-w}{2}. \qquad x := \frac{v+w}{2} \in t^\perp.$$

$$\left.\begin{aligned} v &= t + x. \\ w &= -t + x. \end{aligned}\right\} \quad \left.\begin{aligned} c_0 &:= \|t\|. \\ u &:= t/c_0. \\ t &= c_0 u. \end{aligned}\right\} \quad \left.\begin{aligned} v &= c_0 u + x_0. \\ w &= -c_0 u + x_0. \end{aligned}\right\} \quad x_0 \in u^\perp.$$

$v = c_0 u + x_0.$

$w = -c_0 u + x_0.$

$x_0 \in u^\perp.$

Find orthog $Q_2 \in \mathbb{R}^{3\times 3}$ s.t. $Q_2 v = w.$

$Q_2 := I_3 - 2uu^T.$

$Q_2 u = -u.$

$Q_2 x = x.$

Q_2 is orthogonal.

$$Q_2 v = Q_2 (c_0 u + x_0) = -c_0 u + x_0 = w.$$

$v := \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad a := \|v\|, \quad w := \begin{bmatrix} 5 \\ 0 \end{bmatrix}.$

$$a = \sqrt{3^2 + 4^2} = \sqrt{25} = 5. \qquad w = \begin{bmatrix} 5 \\ 0 \end{bmatrix}.$$

$t := v - w.$

$$t = \begin{bmatrix} 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix} = 2 \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

$c_0 := \|t\|.$

$$c_0 = 2 \cdot \sqrt{1+4} = 2 \cdot \sqrt{5}.$$

$u = t / c_0.$

$$u = \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

$Q_2 := I_3 - 2uu^T.$

$$u = \frac{1}{\sqrt{5}}\begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

$$Q_2 := I_3 - 2uu^T.$$

Soln. $Q_2 = \frac{1}{5}\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}.$

$$uu^T = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}.$$

$$Q_2 = \frac{1}{5}\begin{bmatrix} 5 & \\ & 5 \end{bmatrix} - \frac{1}{5}\begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix}.$$

$$= \frac{1}{5}\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}.$$

END