1. Let X be a standard Normal random variable, i.e., $X \sim N(0,1)$.

   (i) Let $Y = X^2$. Find the p.d.f. of Y and identify its distribution.

   (ii) Let $X_1, X_2, \ldots, X_n$ are i.i.d. standard Normal random variables. Use (i) to find the p.d.f. of $X_1^2 + X_2^2 + \ldots + X_n^2$.

2. Let X_i be the amount of money earned by a food truck on State Street on day i. From past experience, the owner of the cart knows that $\mathbb{E}[X_i] = \$5000$.

   (i) Give the best possible upper bound for the probability that the cart will earn at least $\$7,000$ tomorrow.

   (ii) Answer part (i) again with the extra knowledge that $\mathrm{Var}(X_i) = \$4,500$.

   (iii) Continue to assume that for all i we have $\mathbb{E}[X_i] = \$5,000$ and $\mathrm{Var}(X_i) = \$4,500$. Assuming that the amount earned on any given day is independent of the earning on other days, how many days does the cart have to be on State Street to ensure, with a probability at least 0.95, that the cart's average earnings would be between $\$4,950$ and $\$5,050$.

   (iv) Answer (iii) again, but now under the assumption that there is a dependence between the earnings between consecutive days. Specifically, suppose that $\mathrm{Corr}(X_i, X_{i+1}) = 0.5$ and $\mathrm{Corr}(X_i, X_j) = 0$ for $|i - j| \geq 2$.