

1. Let $X \sim \text{Unif}[4, 10]$, that is, the uniform distribution on the interval $[4, 10]$.
 - (i) $\mathbb{P}(X < 6)$.
 - (ii) $\mathbb{P}(|X - 8| > 1)$.
 - (iii) For $4 \leq t \leq 6$, calculate $\mathbb{P}(X < t | X < 6)$.
2. Suppose the cumulative distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0, & \text{if } x < 1 \\ \frac{1}{3}, & \text{if } 1 \leq x < \frac{4}{3} \\ \frac{1}{2}, & \text{if } \frac{4}{3} \leq x < \frac{3}{2} \\ \frac{3}{4}, & \text{if } \frac{3}{2} \leq x < \frac{9}{5} \\ 1, & \text{if } x \geq \frac{9}{5}. \end{cases}$$

Find the probability mass function of X .

3. Suppose the random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0, & \text{if } x < \sqrt{2} \\ x^2 - 2, & \text{if } \sqrt{2} \leq x < \sqrt{3} \\ 1, & \text{if } x \geq \sqrt{3} \end{cases}$$

- (i) Find the smallest interval $[a, b]$ such that $\mathbb{P}(a \leq X \leq b) = 1$.
- (ii) Find $P(X = 1.6)$.
- (iii) Find $\mathbb{P}(1 \leq X \leq \frac{3}{2})$.
- (iv) Find the probability density function of X .