1. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $U(0,1)$ random variables. Let $U = \min(X_1, X_2, \ldots, X_n)$ and $V = \max(X_1, X_2, \ldots, X_n)$.

   (i) For each $0 < x < y < 1$, find $\mathbb{P}(U > x, V < y)$. Note that $\mathbb{P}(U > x, V < y)$ should depend on x and y.

   (ii) Find the joint p.d.f. of U and V.
   [Hint:
   $$\mathbb{P}(U > x, V < y) = \int_x^1 \int_0^y f_{U,V}(u,v)\,dv\,du,$$
   where $f_{U,V}$ is the joint p.d.f. of U and V.]

2. The life-time of a car is a random variable with p.d.f.
   $$f(x) = \frac{1}{10}e^{-x/10}, \text{ if } x > 0.$$
   The car will be replaced at the end of its life-time or 6 years, whichever comes earlier. Calculate the average time before the car is replaced.

3. There are five closed boxes on a table. Three of the boxes have nice prizes inside. The other two do not. You open boxes one at a time until you find a prize (and then you stop). Let X be the number of boxes you open, including the last one that contains a prize.

   (i) Find the probability mass function of x.

   (ii) Find $\mathbb{E}[X]$.

   (iii) Find $\mathrm{Var}(X)$.

   (iv) Suppose the prize inside each of the three good boxes is \$100, but each empty box you open costs you \$100. What is your expected gain or loss in this game?