

1. Five children A, B, C, D and E play catch. If A has the ball, then he/she is equally likely to throw the ball to B, D or E. If B has the ball, then he/she is equally likely to throw the ball to A, C or E. If either C or E gets the ball, they keep throwing it at each other. If D gets the balls, he/she runs away with it. Find the transition matrix, and classify the states as recurrent or transient.
2. Let $(X_n)_{n \geq 0}$ be a Markov chain on state space $S = \{1, 2, 3\}$ with transition matrix

$$P = \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}.$$

- (a) Compute $\rho_{1,2}$. Recall that $\rho_{x,y} := \mathbf{P}_x(T_y < \infty)$.
- (b) Find $\mathbf{E}_2[N(2)]$, where $N(2) := \sum_{n=1}^{\infty} \mathbf{1}_{\{X_n=2\}}$ is the total number of returns to 2 by the MC.