1. Consider a Markov chain on $S = \{0, 1, 2, \ldots\}$ whose non-zero transition probabilities are as listed below:

$$P(0,1) = 1, \qquad P(m, m+1) = \frac{m}{m+1} \quad \text{and} \quad P(m, 0) = \frac{1}{m+1}, \quad m \geq 1.$$

   Let T_0 be the first return time to state 0.

   (a) Compute $\mathbf{P}_0(T_0 = n)$ for any $n \geq 1$.

   (b) Show that $\mathbf{P}_0(T_0 < \infty) = 1$ and hence 0 is a recurrent state.

   (b) Show that $\mathbf{E}_0[T_0] = \infty$ and hence 0 is a null recurrent state.

2. Find all stationary distributions of the Markov chain with the following transition matrix:

	1	**2**	**3**	**4**	**5**	**6**	**7**
1	0	0	0	1	0	0	0
2	0.1	0	0.3	0	0	0	0.6
3	0	0.7	0	0	0	0	0.3
4	1	0	0	0	0	0	0
5	0	0	0	0	0.2	0.8	0
6	0	0	0	0	0.4	0.6	0
7	0	0.3	0.4	0	0.3	0	0