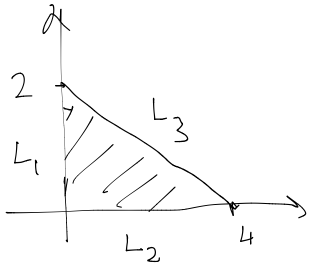
Math 2263 Section 10 Quiz 5

Name: ____________________

Time limit: 15 minutes

1. (7 points) Find the absolute minimum and maximum values of the function $f(x, y) = x + y - xy$ on the closed triangular region with vertices $(0, 0)$, $(0, 2)$, and $(4, 0)$.



$\nabla f(x,y) = \langle 1-y, 1-x \rangle \to$ only crit. pt. is $(1,1)$ - inside the region

$\boxed{f(1,1) = 1}$

Boundaries:

L_1: $x=0$, $0 \le y \le 2 \Rightarrow f(x,y) = y$ $\boxed{\text{min } 0 \quad \text{max } 2}$

L_2: $y=0$, $0 \le x \le 4 \Rightarrow f(x,y) = x$ $\boxed{\text{min } 0 \quad \text{max } 4}$

So abs. min $= 0$

abs. max $= 4$

L_3: $x+2y=4$, $0 \le y \le 2$

$$g(y) := f(x,y) = 4-2y+y-(4-2y)y = 4-y-4y+2y^2 = 2y^2-5y+4$$

$g'(y) = 4y-5 \to$ crit. pt. $y = 5/4$

$\boxed{g(0)=4}$, $\boxed{g(2)=2}$, $\boxed{g(5/4) = \frac{25}{8} - \frac{25}{4} + \frac{16}{4} = \frac{7}{8}}$

2. (8 points) Using Lagrange multipliers, find the extreme values of the function $g(x, y, z) = x + 2y$ subject to the constraints $\underbrace{x + y + 2z}_{h(x,y,z)} = 3$ and $\underbrace{y^2 + z^2}_{k(x,y,z)} = 9$.

$\nabla h = \langle 1,1,2 \rangle$ $\quad \nabla k = \langle 0, 2y, 2z \rangle$

$\nabla g = \langle 1,2,0 \rangle$

Lagrange mult: Solve $\nabla g = \lambda \nabla h + \mu \nabla k$ together with $\boxed{x+y+2z=3}$ (1) and $\boxed{y^2+z^2=9}$ (2)

(3) $\boxed{\begin{aligned} 1 &= \lambda \cdot 1 + \mu \cdot 0 \\ 2 &= \lambda \cdot 1 + \mu \cdot 2y \\ 0 &= \lambda \cdot 2 + \mu \cdot 2z \end{aligned}}$ $\rightsquigarrow$ $\lambda = 1$, $2\mu y = 2 - \lambda = 1$, $2\mu z = -2\lambda = -2$ $\to \frac{y}{z} = \frac{-1}{2} \to z = -2y$

By (1) $9 = y^2 + (-2y)^2 = 5y^2$

Two cases:

i) $y = \frac{3}{\sqrt{5}} \to z = \frac{-6}{\sqrt{5}} \to x \overset{\text{by (1)}}{=} 3 - \frac{3}{\sqrt{5}} + \frac{12}{\sqrt{5}} = 3 + \frac{9}{\sqrt{5}}$

At this point, $g(x,y,z) = \boxed{3 + \frac{15}{\sqrt{5}}}$ ← abs. max

ii) $y = \frac{-3}{\sqrt{5}} \to z = \frac{6}{\sqrt{5}} \to x = 3 + \frac{3}{\sqrt{5}} - \frac{12}{\sqrt{5}} = 3 - \frac{9}{\sqrt{5}}$

Here, $g(x,y,z) = \boxed{3 - \frac{15}{\sqrt{5}}}$ ← abs. min