Name: ____

CSE/MATH 455 Exam II November 15, 1996 Professor Arnold

16 pts 1. Find the Hermite cubic interpolant to $f(x) = \sqrt{x}$ at $x = 1$ and $x = 4$.

$f'(x) = \dfrac{1}{2\sqrt{x}}$. $f(1) = 1,\ f'(1) = 1/2,\ f(4) = 2,\ f'(4) = 1/4.$

The divided difference table is

$$
\begin{array}{ccccc}
1 & 1 & & & \\
 & & f'(1) = \frac{1}{2} & & \\
1 & 1 & & \frac{-1}{18} & \\
 & & \frac{1}{3} & & \frac{1}{108} \\
4 & 2 & & \frac{-1}{36} & \\
 & & f'(4) = \frac{1}{4} & & \\
4 & 2 & & &
\end{array}
$$

Thus Newton's form is

$$p(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{18}(x-1)^2 + \frac{1}{108}(x-1)^2(x-4).$$

26 pts 2. Consider the quadrature rule on $[-1,1]$ given by

$$\int_{-1}^{1} f(x)\,dx \approx \frac{3}{4}f(-2/3) + \frac{1}{2}f(0) + \frac{3}{4}f(2/3).$$

a) What is the degree of precision of this rule?

$\int_{-1}^{1} 1\,dx = 2,\qquad \frac{3}{4}\cdot 1 + \frac{1}{2}\cdot 1 + \frac{3}{4}\cdot 1 \stackrel{?}{=} 2$ ✓

$\int_{-1}^{1} x\,dx = 0,\qquad \frac{3}{4}\cdot\frac{-2}{3} + \frac{1}{2}\cdot 0 + \frac{3}{4}\cdot\frac{2}{3} \stackrel{?}{=} 0$ ✓

$\int_{-1}^{1} x^2\,dx = \frac{2}{3},\qquad \frac{3}{4}\cdot\frac{4}{9} + \frac{1}{2}\cdot 0 + \frac{3}{4}\cdot\frac{4}{9} \stackrel{?}{=} \frac{2}{3}$ ✓

$\int_{-1}^{1} x^3\,dx = 0,\qquad \frac{3}{4}\cdot\frac{-8}{27} + \frac{1}{2}\cdot 0 + \frac{3}{4}\cdot\frac{8}{27} \stackrel{?}{=} 0$ ✓

$\int_{-1}^{1} x^4\,dx = \frac{2}{5},\qquad \frac{3}{4}\cdot\frac{16}{81} + \frac{1}{2}\cdot 0 + \frac{3}{4}\cdot\frac{16}{81} \stackrel{?}{=} \frac{2}{5}$ NO!

The degree of precision is 3.

b) Use scaling to write down the corresponding rule on an arbitrary interval $[a,b]$.

$$\int_a^b f(x)\,dx \approx \frac{b-a}{8}\left[3f\left(\frac{5a+b}{6}\right) + 2f\left(\frac{a+b}{2}\right) + 3f\left(\frac{a+5b}{6}\right)\right]$$

16 pts 3. A certain cubic spline $s(x)$ for the knot sequence $x_0 = -2$, $x_1 = 0$, $x_2 = 1$, interpolates the data $y_0 = -7$, $y_1 = 1$, $y_2 = 4$ at the knots. If $s(x) = 2x^3 + 3x^2 + 2x + 1$ on the interval $[-2, 0]$, what is the formula for $s(x)$ on the interval $[0, 1]$?

From the above formula $s(0) = 1$, $s'(0) = 2$, $s''(0) = 6$. *Writing* $s(x) = a + bx + cx^2 + dx^3$ *on* $[0, 1]$, *we get* $a = 1$, $b = 2$, *and* $c = 3$, *so* $s(x) = 1 + 2x + 3x^2 + dx^3$. *Since* $s(1) = 4$, $d = -2$. *Thus*

$$s(x) = 1 + 2x + 3x^2 - 2x^3 \qquad \textit{on } [0, 1].$$

14 pts 4. When using the composite Simpson rule, halving the size of the subintervals typically reduces the error by about what factor?

a) 2

b) 4

c) 8

d) 16 ✓

14 pts 5. Consider a cubic spline with knots $x_0 < x_1 < \cdots < x_n$ interpolating the values y_0, y_1, ..., y_n at the knots. Which of the following is true:

a) Given the choice of knots and values, there is a unique interpolating cubic spline.

b) Given the choice of knots and values, there is more than one interpolating cubic spline. ✓ *(we need end conditions to specify it uniquely)*

c) There is a unique interpolating cubic spline only if $n = 3$.

d) There may be an interpolating cubic spline or there may not be one, depending on the particular points and values.

14 pts 6. Which of the following expressions *does NOT exactly equal* the error in the simple trapezoidal rule on $[a, b]$?

a) $\displaystyle\int_a^b f[a,b,x](x-a)(x-b)\,dx$

b) $\displaystyle -\frac{1}{12}f''(\eta)(b-a)^3$ for some $\eta \in [a,b]$

c) $\displaystyle -\frac{(b-a)^2}{12}[f'(b)-f'(a)]$ ✓ *(the endpoint corrections are only an approximation)*

d) $\displaystyle\int_a^b \frac{(x-a)(x-b)}{2} f''(x)\,dx$