Name: ____

CSE/MATH 455 Final Exam December 10, 1996 Professor Arnold

Do 10 of the 11 problems and omit one. Write the word "OMIT" by the problem you are omitting (or I will omit the last problem). All problems are worth 10 points. The last four are multiple choice.

1. If y approximates some positive number x with relative error at most 4×10^{-6}, how large is the relative error of $\sqrt{y}$ as an approximation of $\sqrt{x}$? Show your derivation as well as your result.

2. The function

$$f(x) = [\cos(x^2 e^{-x}) - 0.92]e^x + \sin x + x^2 - 4x + \pi$$

satisfies $f(1) = 1.02\ldots$ and $f(3) = -0.093\ldots$. How many additional evaluations of f are needed to find an interval of length at most 0.001 which contains a root of f, using the bisection method?

3. a) Suppose we approximate $\sqrt{7}$ by using the secant method to solve $x^2 - 7 = 0$. Give the formula for x_{n+1} in terms of x_n and x_{n-1}. (Simplify the formula as much as possible.)

b) Starting with $x_0 = 2$ and $x_1 = 3$ find x_2.

4. a) Define what it means to say that a sequence of numbers x_n is *converging linearly* to a number α.

b) Assuming that x_n is converging linearly to some (unknown) number α, derive Aitken's formula for combining x_{n-1}, x_n, and x_{n+1} to obtain an improved approximation to α. (Show both the derivation and the formula.)

5. a) Write out in full the expression giving the approximation to $\int_2^5 \cos x^3 \, dx$ using Simpson's rule with equally spaced points and 7 evaluations of the integrand.

b) If the approximation is made with 31 function evaluations instead of 7 by about what factor will the error decrease?

6. Find a lower triangular matrix L and and upper triangular matrix U so that

$$LU = \begin{pmatrix} 5 & 3 & 0 \\ 1 & -9 & 2 \\ 4 & 0 & 2 \end{pmatrix}.$$

7. An upper Hessenberg matrix is an $n \times n$ matrix for which the entries $a_{ij} = 0$ whenever $i > j + 1$. In other words, a matrix for which the nonzero entries occurs in the positions marked with an $\times$:

$$\begin{pmatrix} \times & \times & \times & \dots & \times & \times \\ \times & \times & \times & \dots & \times & \times \\ 0 & \times & \times & \dots & \times & \times \\ 0 & 0 & \times & \dots & \times & \times \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \times & \times \end{pmatrix}$$

a) Write the algorithm for reducing an upper Hessenberg matrix to triangular form by Gaussian elimination without pivoting.

b) How many operations does your algorithm take (count multiplications and divisions as one operation and ignore additions).

8. If $s(x)$ is a cubic spline interpolating $f(x)$ at the knots $x_0, x_1, \dots, x_n$, then $s''(x)$ is:

   a) a continuous piecewise linear function interpolating $f(x)$ at the knots.

   b) a continuous piecewise linear function interpolating $f''(x)$ at the knots.

   c) a continuous piecewise linear function which in general does not interpolate either $f(x)$ or $f''(x)$ at the knots

   d) in general not a continuous piecewise linear function

9. Which is a valid criticism of the Gaussian quadrature rules?

   a) For large number of points the weights are negative, and this contributes to amplification of function evaluation errors.

   b) The Gaussian rules require evaluation of function derivatives as well as function values.

   c) If a Gaussian rule is applied on an interval and then the interval is bisected and the rule applied to each half, no function evaluations can be shared by the two computations.

   d) Other quadrature rules achieve a higher degree of precision for the same number of function evaluations.

10. Consider the decomposition of a matrix as LL^T where L is a nonsingular lower triangular matrix. Which of the following is true?

    a) Any nonsingular matrix can be factored in this way.

    b) Not every nonsingular matrix can be factored in this way, but only those that have nonzero diagonal matrices.

    c) Not every nonsingular matrix can be factored in this way, but for every nonsingular matrix its rows may be permuted so that the resulting matrix can be factored in this way.

    d) Only positive definite matrices can be factored in this way.

11. Which of the following mathematically equivalent expressions is the most suitable form for evaluation in floating point arithmetic, assuming x is near $\pi/2$?

a) $\sec x - \tan x$

b) $\dfrac{1-\sin x}{\cos x}$

c) $\dfrac{1-\sin^2 x}{\cos x(1+\sin x)}$

d) $\dfrac{\cos x}{1+\sin x}$

Enjoy the break!