CSE/MATH 455 Homework 4 Arnold

Numerical Integration

1. Compute $\displaystyle\int_2^5 \frac{1}{1+5xe^{-x^2}}\,dx$ to with relative error less than 10^{-8} using the composite Simpson's rule. Give the exact value you obtain (showing all digits) and the number of function evaluations you use.

2. Consider the simple quadrature rule on $[0,1]$ derived as

$$\int_0^1 f(x)\,dx \approx \int_0^1 p(x)\,dx$$

where $p(x)$ is the linear polynomial interpolating $f(x)$ at $x=0$ and $x=3/4$.

a) Write down this quadrature rule in the usual form (a weighted sum of values of $f(x)$).

b) Scale this quadrature rule to obtain a simple quadrature rule approximation to $\int_a^b f(x)\,dx$, where $[a,b]$ is an arbitrary interval.

c) Write down the approximation to $\int_a^b f(x)\,dx$ obtained by applying this rule as a composite rule with n equal subintervals.

3. a) Obtain a quadrature rule on $[0,1]$ as above, but using the linear polynomial interpolating $f(x)$ at $x=0$ and $x=c$, where, for the moment, c is an unspecified point in $[0,1]$.

b) Is there any value of c for which this rule is exact for all quadratic polynomials $f(x)$?

c) Is there any value of c for which this rule is exact for all cubic polynomials $f(x)$?

d) For the best value of c (as determined in b and c above), write down the simple quadrature rule scaled to an arbitrary interval.