Name: ______________________________ Section: __________

ID#: ______________________________

CSE/MATH 455, Arnold | Lab 5 | October 31, 1996

Some Quadrature Rules with Equally Spaced Points

Files to copy: `trapez.m`, `f.m`, and `showtrapez.m` from `~dna2/pub/quadrature`.

In this lab we will investigate some quadrature rules with equally spaced points:

(1) **Trapezoidal rule**

$$\int_a^b f(x)\,dx \approx \frac{h}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)];$$

(2) **Corrected trapezoidal rule**

$$\int_a^b f(x)\,dx \approx \frac{h}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)] - \frac{h^2}{12}[f'(b) - f'(a)];$$

(3) **Simpson's rule**

$$\int_a^b f(x)\,dx \approx \frac{h}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)].$$

Here $h = (b-a)/n$, $x_j = a + jh$, and for Simpson's rule n is even.

We will use these rules for the integrals

a) error function: $\displaystyle\int_0^1 e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}\,\mathrm{erf}(1)$; b) singular integrand: $\displaystyle\int_0^2 \sqrt{x}e^{-x^2}\,dx$.

The first integral is the error function and you can find the exact value for comparison with the command `erf(1)` in Matlab. The second integrand is a not a C^1 function because its derivative is undefined at $x = 0$ (check this!). To get an "exact value" for comparison, use Matlab's high order adaptive integrator `exact=quad8('f',0,2,1.e-12)`.

1. Use `showtrapez` to study the accuracy of the trapezoidal rule for the error function integral. What value of n is needed to obtain a relative error of 10^{-4}? __________ What is the rate of convergence of the trapezoidal rule for this example? __________

2. Change `showtrapez` to handle the second integral above. Now what value of n is needed to obtain a relative error of 10^{-4}? __________ What is the rate of convergence of the trapezoidal rule for this example? __________

3. Modify `trapez.m` to make a new function `trapezc('f',a,b,fpa,fpb,n)` which computes the corrected trapezoidal rule. Note that the user must supply `fpa` $= f'(a)$ and `fpb` $= f'(b)$. The corrected trapezoidal rule cannot be applied to the second integral because $f'(0)$ does not exist.

n needed to achieve relative error of 10^{-4} for first integral __________

rate of convergence of corrected trapezoidal rule for first integral __________

4. Now make the same study for Simpson's rule.

n needed to achieve relative error of 10^{-4} for first integral __________

rate of convergence of Simpson's rule for first integral __________

n needed to achieve relative error of 10^{-4} for second integral __________

rate of convergence of Simpson's rule for second integral __________

For both examples, compare the error for the trapezoidal rule using 129 evaluations of f and the error for Simpson's rule using the same 129 evaluations.

5. Study the convergence for the integral $\int_0^\pi \sqrt{1+\sin^4(x)}\,dx$. Try to explain what you find.

6. Study the midpoint rule as well. Which seems to be more accurate, the trapezoidal rule or Simpson's rule?