Name: _____

CSE/MATH 455 Exam I October 10, 1997 Professor Arnold

Do 5 of the 6 problems and omit 1. State which problem you omit.
Work neatly and carefully. You must *show your work* to get credit.

1. Recall that IEEE single precision uses a 24 bit mantissa and an exponent running from -126 to 127. What is the distance from $9\frac{1}{2}$ to the next larger IEEE single precision floating point number?

2. The function $f(x) = \cos(2\pi x^2) - e^x + 1$ satisfies $f(0) = 1$ and $f(1) = 2 - e \approx -0.71828$. Using the bisection method, how many additional evaluations of the function are needed to find an interval of length at most 0.001 which contains a root of the function?

3. Give an example of numbers x and y and approximations $\bar{x}$ and $\bar{y}$ which each have relative error less than 1%, but so that $\bar{x}+\bar{y}$ has relative error of more than 50% as an approximation to $x+y$.

4. Recall that $\cos \pi/4 = \sqrt{2}/2$. If x is an approximation to $\pi/4$ with a relative error of about 0.01, then about what is the relative error of $\cos x$ as an approximation to $\sqrt{2}/2$? Explain how you get your answer.

5. Consider the equation $x = e^x - 5$. Sketch a graph to show that there is a unique positive solution x^* this equation.

   a) Write down an algorithm based on Newton's method to solve this equation.

   b) For what positive starting values will this algorithm converge to x^*? Justify your answer.

6. Consider again the equation $x = e^x - 5$, but now use the iteration $x_{n+1} = e^{x_n} - 5$. For what positive values of x_0 will this algorithm converge to x^*? Justify your answer.